Reteaching (continued)

The Polygon Angle-Sum Theorems

Exercises

Find the sum of the interior angles of each polygon.

1. quadrilateral **360**   **2.** octagon **1080**   **3.** 18-gon **2880**

4. decagon **1440**   **5.** 12-gon **1800**   **6.** 28-gon **4680**

Find the measure of an interior angle of each regular polygon. Round to the nearest tenth if necessary.

7. decagon **144**   **8.** 12-gon **150**   **9.** 16-gon **157.5**

**10.** 24-gon **165**   **11.** 32-gon **168.8**   **12.** 90-gon **176**

Exterior Angles of a Polygon

The exterior angles of a polygon are those formed by extending sides. There is one exterior angle at each vertex.

Polygon Exterior Angle-Sum Theorem:

The sum of the measures of the exterior angles of a polygon is 360.

A pentagon has five exterior angles. The sum of the measures of the exterior angles is always 360, so each exterior angle of a regular pentagon measures 72.
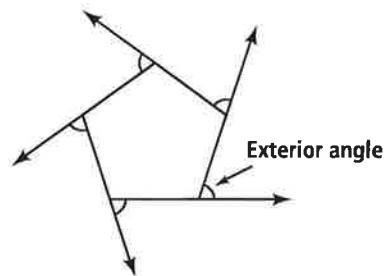

Exterior angle

Exercises

Find the measure of an exterior angle for each regular polygon. Round to the nearest tenth if necessary.

13. octagon **45**   **14.** 24-gon **15**   **15.** 34-gon **10.6**

16. decagon **36**   **17.** heptagon **51.4**   **18.** hexagon **60**

19. 30-gon **12**   **20.** 28-gon **12.9**   **21.** 36-gon **10**

22. Draw a Diagram A triangle has two congruent angles, and an exterior angle that measures 140. Find two possible sets of angle measures for the triangle. Draw a diagram for each. **40, 40, 100; 40, 70, 70**
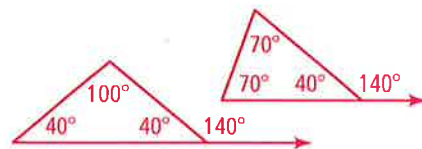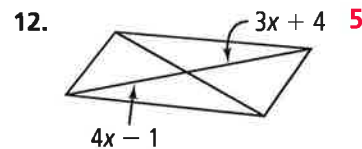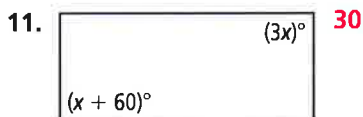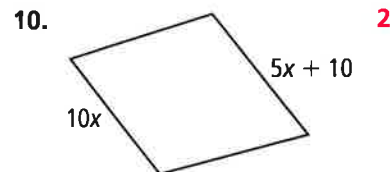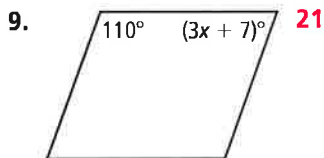
Reteaching (continued)

Properties of Parallelograms

Exercises

Find the value of x in each parallelogram.

1. **50**

   $x°$ $130°$

2. **2**

   $5x + 2$ $6x$

3. **35**

   $35°$ $x°$

4. **7**

   $3x - 2$ 19

5. **4**

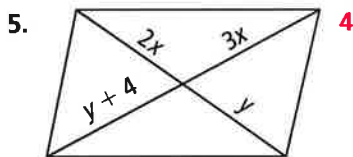 $2x$ $3x$ $y + 4$ y

6. **3**

   $3x - 1$ y $x + 3$ $y + 2$

7. **8**

   $3x - 2$ $2x - 4$ $x + 4$ $2x + 6$

8. **40**

   $(x + 35)°$ $(2x + 25)°$ $(3x - 15)°$ $(2x - 5)°$

9. **21**

   $110°$ $(3x + 7)°$

10. **2**

    $5x + 10$ $10x$

11. **30**

    $(3x)°$ $(x + 60)°$

12. **5**

    $3x + 4$ $4x - 1$
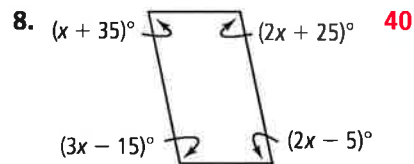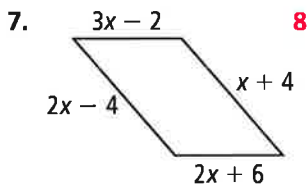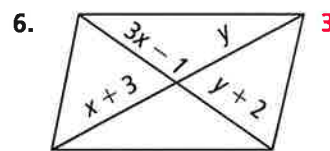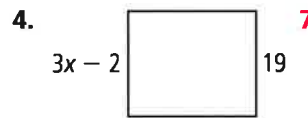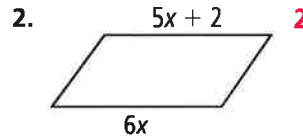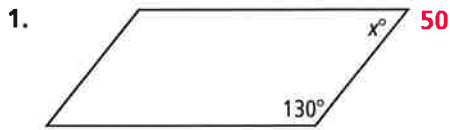
13. **Writing** Write a statement about the consecutive angles of a parallelogram.
    Consecutive angles of a parallelogram are supplementary.

14. **Writing** Write a statement about the opposite angles of a parallelogram.
    Opposite angles of a parallelogram are congruent.

15. **Reasoning** One angle of a parallelogram is 47. What are the measures of the other three angles in the parallelogram?
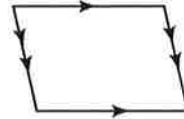   **47, 133, and 133**

Reteaching

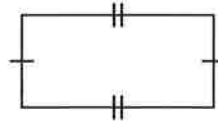## Proving That a Quadrilateral Is a Parallelogram

Is a quadrilateral a parallelogram?

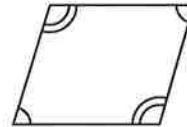There are five ways that you can confirm that a quadrilateral is a parallelogram.

If both pairs of opposite sides are parallel,
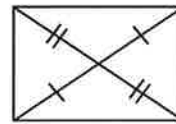then the quadrilateral is a parallelogram.

If both pairs of opposite sides are congruent,
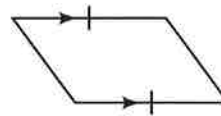then the quadrilateral is a parallelogram.

If both pairs of opposite angles are congruent,
then the quadrilateral is a parallelogram.

If the diagonals bisect each other, then the
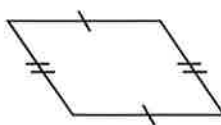quadrilateral is a parallelogram.
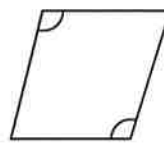
If one pair of sides is both congruent and parallel,
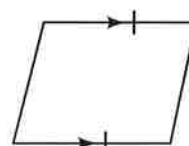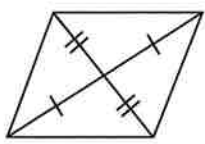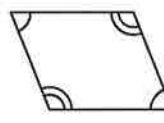then the quadrilateral is a parallelogram.

Exercises

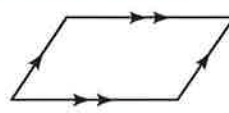**Can you prove that the quadrilateral is a parallelogram based on the given information? Explain.**

1.

yes; opposite sides ≅

2.

no; not enough info

3.

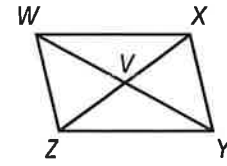yes; 1 pair of sides ≅ and ∥

4.

Yes; diagonals bisect each other.

5.

yes; opposite ∠s ≅

6.

yes; opposite sides ∥

Reteaching (continued)

Proving That a Quadrilateral Is a Parallelogram

Determine whether the given information is sufficient to prove that quadrilateral *WXYZ* is a parallelogram.
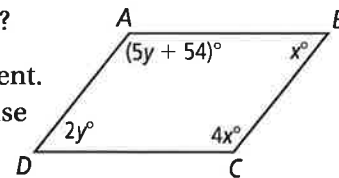
7. $\overline{WY}$ bisects $\overline{ZX}$ **no**

8. $\overline{WX} \parallel \overline{ZY}; \overline{WZ} \cong \overline{XY}$ **no**

9. $\overline{VZ} \cong \overline{VX}; \overline{WX} \cong \overline{YZ}$ **no**

10. $\angle VWZ \cong \angle VYX; \overline{WZ} \cong \overline{XY}$ **yes**

You can also use the requirements for a parallelogram to solve problems.

Problem

For what value of *x* and *y* must figure *ABCD* be a parallelogram?

In a parallelogram, the two pairs of opposite angles are congruent. So, in *ABCD*, you know that $x = 2y$ and $5y + 54 = 4x$. You can use these two expressions to solve for *x* and *y*.

Step 1: Solve for *y*.

$5y + 54 = 4x$	
$5y + 54 = 4(2y)$	Substitute 2*y* for *x*.
$5y + 54 = 8y$	Simplify.
$54 = 3y$	Subtract 5*y* from each side.
$18 = y$	Divide each side by 3.

Step 2: Solve for *x*.

$x = 2y$	Opposite angles of a parallelogram are congruent.
$x = 2(18)$	Substitute 18 for *y*.
$x = 36$	Simplify.

For *ABCD* to be a parallelogram, *x* must be 36 and *y* must be 18.

Exercises

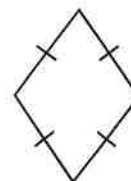For what value of *x* must the quadrilateral be a parallelogram?

11. **18**

12. **3**

13. **3**

14. **8**
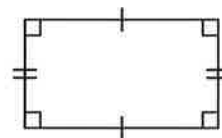
15. **14**

16. **5.5**

Reteaching

Properties of Rhombuses, Rectangles, and Squares

Rhombuses, rectangles, and *squares* share some characteristics. But they also have some unique features.
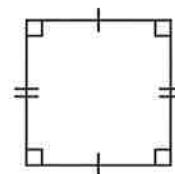
A rhombus is a parallelogram with four congruent sides.

A rectangle is a parallelogram with four congruent angles. These angles are all right angles.

A square is a parallelogram with four congruent sides and four congruent angles. A square is both a rectangle and a rhombus. A square is the only type of rectangle that can also be a rhombus.

Here is a Venn diagram to help you see the relationships.
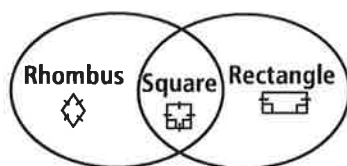
There are some special features for each type of figure.

Rhombus: The diagonals are perpendicular.
The diagonals bisect a pair of opposite angles.

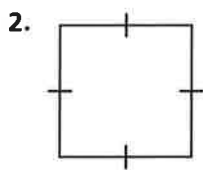**Rectangles:** The diagonals are congruent.

Squares: The diagonals are perpendicular.
The diagonals bisect a pair of opposite angles (forming two 45°
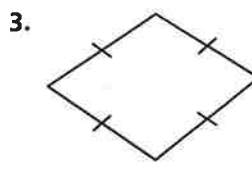angles at each vertex).
The diagonals are congruent.

Exercises

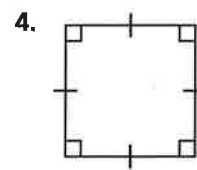**Decide whether the parallelogram is a rhombus, a rectangle, or a square.**

1.
rectangle

2.
rhombus

3.
rhombus
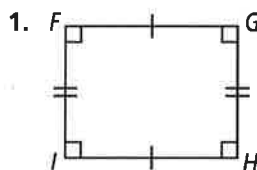
4.
square

Reteaching (continued)

Properties of Rhombuses, Rectangles, and Squares

List the quadrilaterals that have the given property. Choose among
parallelogram, rhombus, rectangle, **and** *square.*

5. Opposite angles are supplementary.
 rectangle, square

6. Consecutive sides are $\cong$.
 rhombus, square

7. Consecutive sides are $\perp$.
 rectangle, square

8. Consecutive angles are $\cong$.
 rectangle, square

You can use the properties of rhombuses, rectangles, and squares to
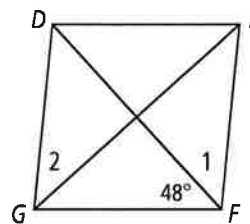solve problems.

Problem

Determine the measure of the numbered angles in rhombus *DEFG.*

$\angle 1$ is part of a bisected angle. $m\angle DFG = 48$, so $m\angle 1 = 48$.

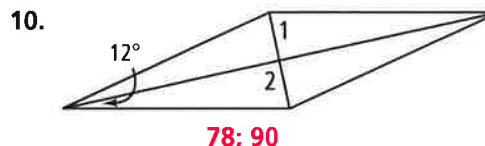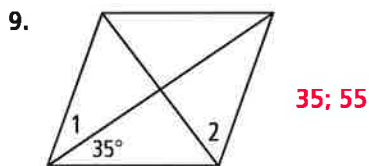Consecutive angles of a parallelogram are supplementary.
$m\angle EFG = 48 + 48 = 96$, so $m\angle DGF = 180 - 96 = 84$.

The diagonals bisect the vertex angle, so $m\angle 2 = 84 \div 2 = 42$.
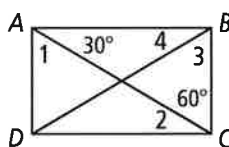
Exercises

Determine the measure of the numbered angles in each rhombus.

9.

35; 55

10.

78; 90

Determine the measure of the numbered angles in each figure.

11. rectangle *ABCD* **60; 30; 60; 30**

12. square *LMNO* **90; 90; 45**
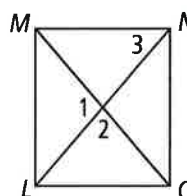
Algebra *TUVW* **is a rectangle. Find the value of** *x* **and the length of**
each diagonal.

13. $TV = 3x$ and $UW = 5x - 10$
 5; 15; 15

14. $TV = 2x - 4$ and $UW = x + 10$
 14; 24; 24

15. $TV = 6x + 4$ and $UW = 4x + 8$
 2; 16; 16

16. $TV = 7x + 6$ and $UW = 9x - 18$
 12; 90; 90

17. $TV = 8x - 2$ and $UW = 5x + 7$
 3; 22; 22

18. $TV = 10x - 4$ and $UW = 3x + 24$
 4; 36; 36

Reteaching

Conditions for Rhombuses, Rectangles, and Squares

A parallelogram is a rhombus if either of these conditions is met:
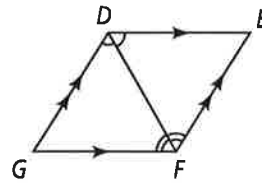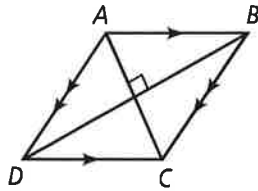
1) The diagonals of the parallelogram are perpendicular. (Theorem 53)

2) A diagonal of the parallelogram bisects a pair of opposite angles. (Theorem 54)

A parallelogram is a rectangle if the diagonals of the parallelogram are congruent.

$\overline{WY} \cong \overline{XZ}$
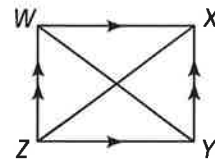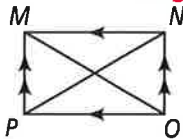

Exercises

Classify each of the following parallelograms as a *rhombus*, a *rectangle*, or a *square*. For each, explain.

1. $\overline{MO} \cong \overline{PN}$ **Rectangle; the diagonals are ≅.**

2.
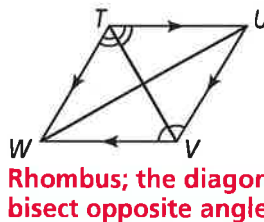
Rhombus; the diagonals bisect opposite angles.

3. $\overline{AC} \cong \overline{BD}$ **Square; the diagonals are ≅ and ⊥.**
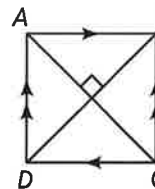
Use the properties of rhombuses and rectangles to solve problems.

Problem

For what value of x is $\square DEFG$ a rhombus?

In a rhombus, diagonals bisect opposite angles.

So, $m\angle GDF = m\angle EDF$.

$(4x + 10) = (5x + 6)$	Set angle measures equal to each other.
$10 = x + 6$	Subtract $4x$ from each side.
$4 = x$	Subtract 6 from each side.

Reteaching (continued)

Conditions for Rhombuses, Rectangles, and Squares

Exercises

4. For what value of x is $\square WXYZ$ a rhombus? **11**

W, X, 22°, 2x°, Z, Y

5. $SQ = 14$. For what value of x is $\square PQRS$ a rectangle?

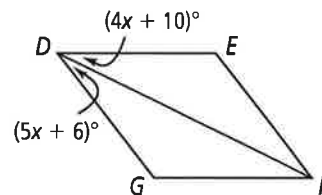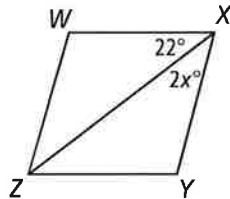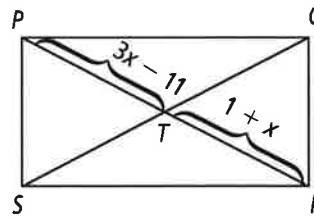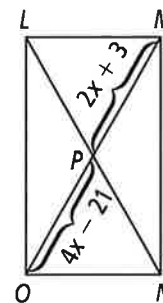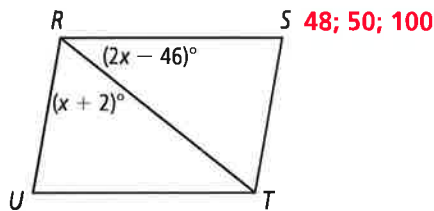Solve for PT. Solve for PR. **6; 7; 14**

P, Q, 3x − 11, 1 + x, T, S, R

6. For what value of x is $\square RSTU$ a rhombus? What is $m\angle SRT$? What is $m\angle URS$?

R, S **48; 50; 100**, $(2x − 46)°$, $(x + 2)°$, U, T

7. $LN = 54$. For what value of x is $\square LMNO$ a rectangle? **12**

L, M, 2x + 3, P, 4x − 21, O, N

8. Given: $\square ABCD$, $\overline{AC} \perp \overline{BD}$ at E.

Prove: $ABCD$ is a rhombus.

Statements	Reasons
1) $\overline{AE} \cong \overline{CE}$	1) __?__ **Diagonals of a $\square$ bisect each other.**
2) $\overline{AC} \perp \overline{BD}$ at E	2) __?__ **Given**
3) __?__ **$\angle AED$ and $\angle CED$ are right angles.**	3) Definition of perpendicular lines
4) __?__ **$\angle AED \cong \angle CED$**	4) __?__ **All right angles are congruent.**
5) __?__ **$\overline{DE} \cong \overline{DE}$**	5) Reflexive Property of Congruence
6) $\triangle AED \cong \triangle CED$	6) __?__ **SAS Postulate**
7) $\overline{AD} \cong \overline{CD}$	7) __?__ **CPCTC**
8) __?__ **$\overline{AB} \cong \overline{CD}, \overline{AD} \cong \overline{BC}$**	8) Opposite sides of a $\square$ are $\cong$.
9) __?__ **$\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$**	9) __?__ **Transitive Property of Congruence**
10) $ABCD$ is a rhombus.	10) __?__ **Definition of rhombus**

Reteaching

Trapezoids and Kites

A *trapezoid* is a quadrilateral with exactly one pair of parallel sides. The two parallel sides are called *bases*. The two nonparallel sides are called *legs*.
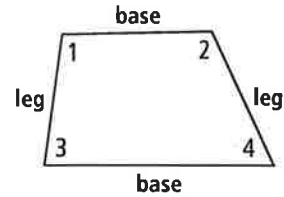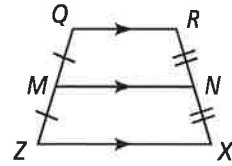
A pair of base angles share a common base.

$\angle 1$ and $\angle 2$ are one pair of base angles.

$\angle 3$ and $\angle 4$ are a second pair of base angles.

In any trapezoid, the *midsegment* is parallel to the bases. The length of the midsegment is half the sum of the lengths of the bases.

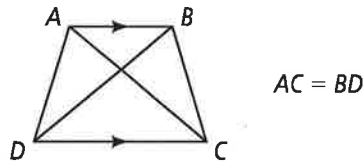$$MN = \tfrac{1}{2}(QR + ZX)$$

An *isosceles trapezoid* is a trapezoid in which the legs are congruent. An isosceles trapezoid has some special properties:

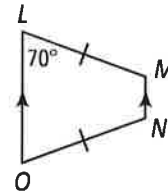Each pair of base angles is congruent.   The diagonals are congruent.
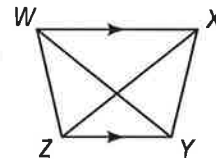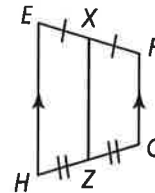
$AC = BD$
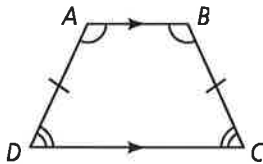
Exercises

1. In trapezoid *LMNO*, what is the measure of $\angle OLM$? **70**
   What is the measure of $\angle LMN$? **110**

2. *WXYZ* is an isosceles trapezoid and $WY = 12$. What is XZ? **12**

3. $\overline{XZ}$ is the midsegment of trapezoid *EFGH*. If $FG = 8$ and $EH = 12$, what is XZ? **10**

Reteaching (continued)

Trapezoids and Kites

A *kite* is a quadrilateral in which two pairs of consecutive sides are congruent and no opposite sides are congruent.

In a kite, the diagonals are perpendicular. The diagonals look like the crossbars in the frame of a typical kite that you fly.

Notice that the sides of a kite are the hypotenuses of four right triangles whose legs are formed by the diagonals.
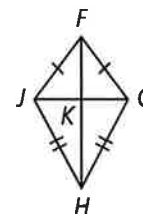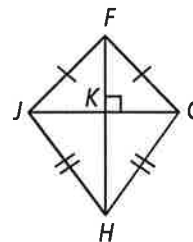
Problem

Write a two-column proof to identify three pairs of congruent triangles in kite *FGHJ*.

Statements	Reasons
1) $m\angle FKG = m\angle GKH = m\angle HKJ = m\angle JKF = 90$	1) Theorem 59
2) $\overline{FG} \cong \overline{FJ}$	2) Given
3) $\overline{FK} \cong \overline{FK}$	3) Reflexive Property of Congruence
4) $\triangle FKG \cong \triangle FKJ$	4) HL Theorem
5) $\overline{JK} \cong \overline{KG}$	5) CPCTC
6) $\overline{KH} \cong \overline{KH}$	6) Reflexive Property of Congruence
7) $\triangle JKH \cong \triangle GKH$	7) SAS Postulate
8) $\overline{JH} \cong \overline{GH}$	8) Given
9) $\overline{FH} \cong \overline{FH}$	9) Reflexive Property of Congruence
10) $\triangle FJH \cong \triangle FGH$	10) SSS Postulate

So $\triangle FKG \cong \triangle FKJ$, $\triangle JKH \cong \triangle GKH$, and $\triangle FJH \cong \triangle FGH$.

Exercises

In kite *FGHJ* in the problem, $m\angle JFK = 38$ and $m\angle KGH = 63$. Find the following angle and side measures.

4. $m\angle FKJ$ **90**

5. $m\angle FJK$ **52**

6. $m\angle FKG$ **90**

7. $m\angle KFG$ **38**

8. $m\angle FGK$ **52**

9. $m\angle GKH$ **90**

10. $m\angle KHG$ **27**

11. $m\angle KJH$ **63**

12. $m\angle JHK$ **27**

13. If $FG = 4.25$, what is JF? **4.25**

14. If $HG = 5$, what is JH? **5**

15. If $JK = 8.5$, what is GJ? **17**