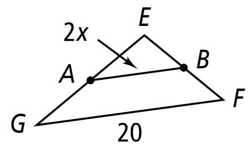
Name ______________________ Class __________ Date __________

Reteaching (continued)

Midsegments of Triangles

Problem

$\overline{AB}$ is a midsegment of $\triangle GEF$. What is the value of x?



$$2AB = GF$$

$$2(2x) = 20$$

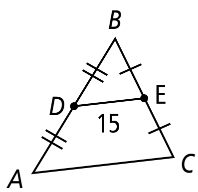
$$4x = 20$$

$$x = 5$$

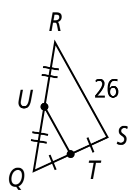
Exercises

Find the length of the indicated segment.

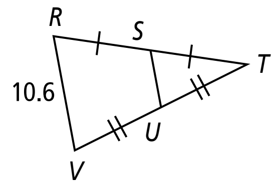
1. AC 30



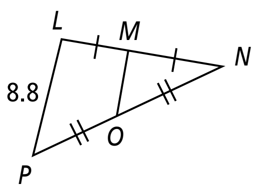
2. TU 13



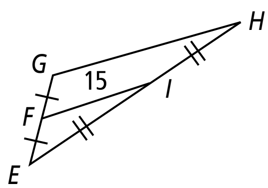
3. SU 5.3



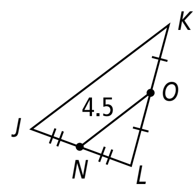
4. MO 4.4



5. GH 30

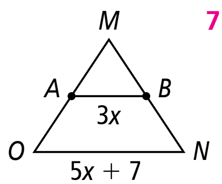


6. JK 9

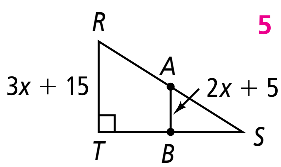


Algebra In each triangle, $\overline{AB}$ is a midsegment. Find the value of x.

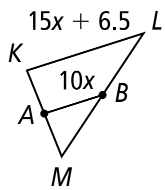
7. 7



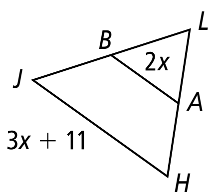
8. 5



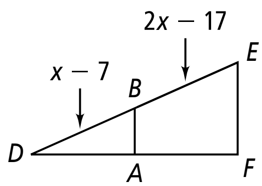
9. 1.3



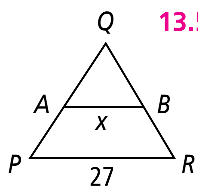
10. 11



11. 10



12. 13.5



Name ______________________ Class ____________ Date ____________

Reteaching

Perpendicular and Angle Bisectors

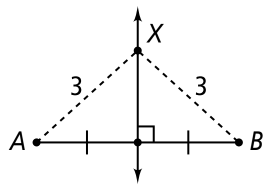
Perpendicular Bisectors

There are two useful theorems to remember about perpendicular bisectors.

Perpendicular Bisector Theorem

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

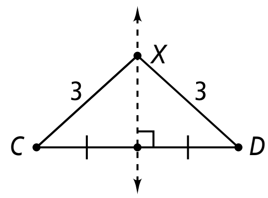
X is on the perpendicular bisector, so it is equidistant from the endpoints *A* and *B*.



Converse of the Perpendicular Bisector Theorem

If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

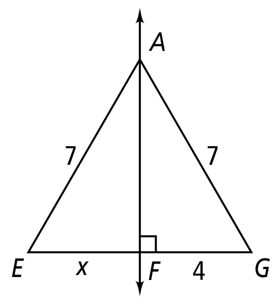
Because *X* is equidistant from the endpoints *C* and *D,* it is on the perpendicular bisector of the segment.



Problem

What is the value of *x?*

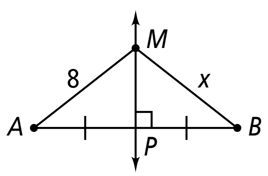
Since *A* is equidistant from the endpoints of the segment, it is on the perpendicular bisector of $\overline{EG}$. So, $EF = GF$ and $x = 4$.



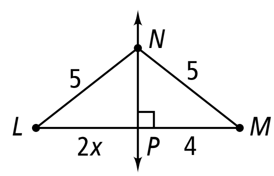
Exercises

Find the value of *x*.

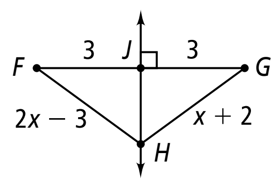
1. 8



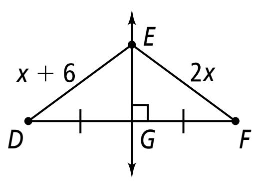
2. 2



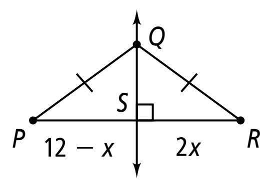
3. 5



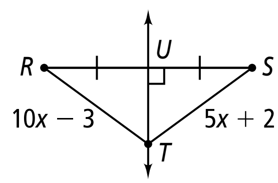
4. 6



5. 4



6. 1



Name ______________________ Class ____________ Date ____________

Reteaching (continued)

Perpendicular and Angle Bisectors

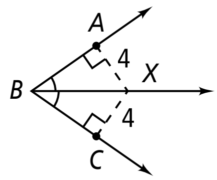
Angle Bisectors

There are two useful theorems to remember about angle bisectors.

Angle Bisector Theorem

If a point is on the bisector of an angle, then the point is equidistant from the sides of the angle.

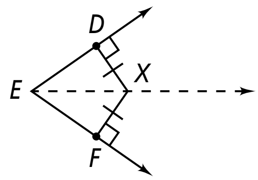
X is on the angle bisector and is therefore equidistant from the sides of the angle.



Converse of the Angle Bisector Theorem

If a point in the interior of an angle is equidistant from the sides of an angle, then the point is on the angle bisector.

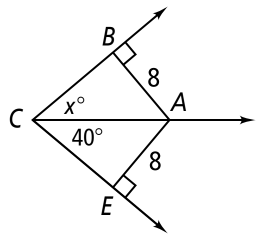
Because *X* is in the interior of the angle and is equidistant from the sides, *X* is on the angle bisector.



Problem

What is the value of *x*?

Because point *A* is in the interior of the angle and it is equidistant from the sides of the angle, it is on the bisector of the angle.



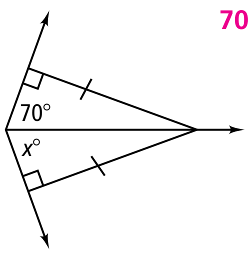
$$\angle BCA \cong \angle ECA$$

$$x = 40$$

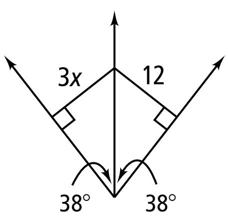
Exercises

Find the value of *x*.

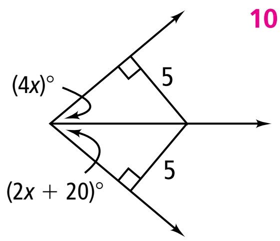
7. 70



8. 4



9. 10

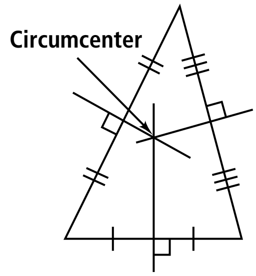


Name ______________________ Class ____________ Date ____________

Reteaching

Bisectors in Triangles

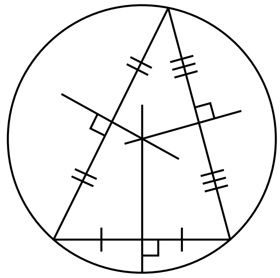
The Circumcenter of a Triangle



If you construct the perpendicular bisectors of all three sides of a triangle, the constructed segments will all intersect at one point. This point of concurrency is known as the circumcenter of the triangle.

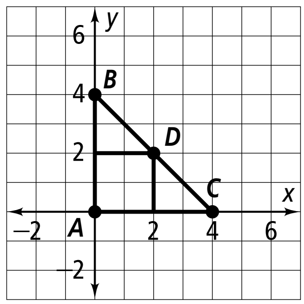
It is important to note that the circumcenter of a triangle can lie inside, on, or outside the triangle.

The circumcenter is equidistant from the three vertices. Because of this, you can construct a circle centered on the circumcenter that passes through the triangle's vertices. This is called a *circumscribed* circle.



Problem

Find the circumcenter of right $\triangle ABC$.



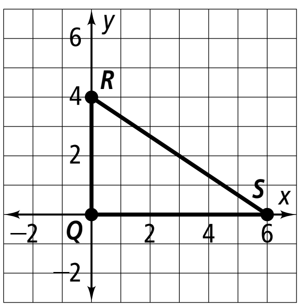
First construct perpendicular bisectors of the two legs, $\overline{AB}$ and $\overline{AC}$. These intersect at (2, 2), the circumcenter.

Notice that for a right triangle, the circumcenter is on the hypotenuse.

Exercises

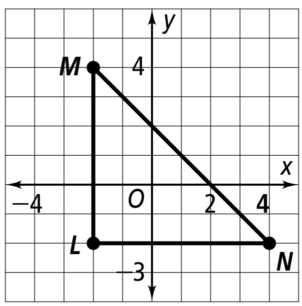
Coordinate Geometry **Find the circumcenter of each right triangle.**

1.



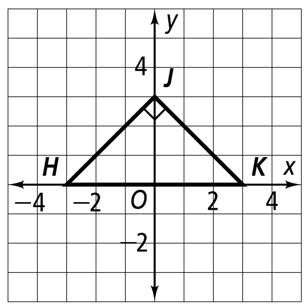
(3, 2)

2.



(1, 1)

3.



(0, 0)

Coordinate Geometry **Find the circumcenter of $\triangle ABC$.**

4. $A(0, 0)$ **(5, 4)**
$B(0, 8)$
$C(10, 8)$

5. $A(-7, 3)$ **(1, −2)**
$B(9, 3)$
$C(-7, -7)$

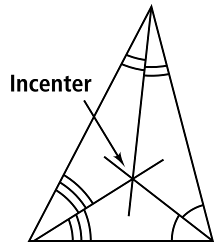
6. $A(-5, 2)$ **(−1, 4)**
$B(3, 2)$
$C(3, 6)$

Reteaching (continued)

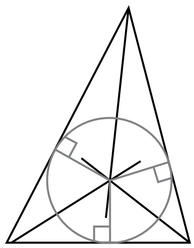
Bisectors in Triangles

The Incenter of a Triangle

If you construct angle bisectors at the three vertices of a triangle, the segments will intersect at one point. This point of concurrency where the angle bisectors intersect is known as the *incenter of the triangle.*



It is important to note that the incenter of a triangle will always lie inside the triangle.



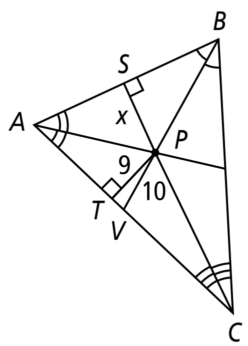
The incenter is equidistant from the sides of the triangle. You can draw a circle centered on the incenter that just touches the three sides of the triangle. This is called an *inscribed* circle.

Problem

Find the value of x.

The angle bisectors intersect at P. The incenter P is equidistant from the sides, so $SP = PT$. Therefore, $x = 9$.

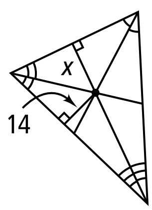
Note that $\overline{PV}$, the continuation of the angle bisector, is not the correct segment to use for the shortest distance from P to $\overline{AC}$.



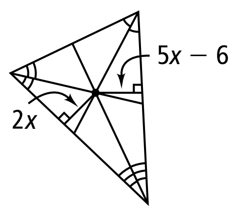
Exercises

Find the value of x.

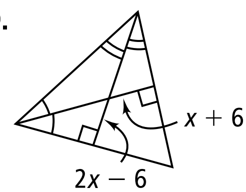
7. 14



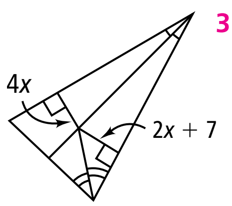
8. 2



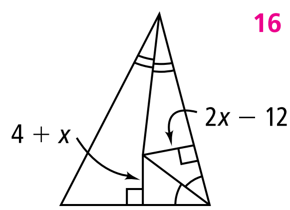
9. 12



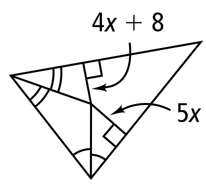
10. 3.5



11. 16



12. 8

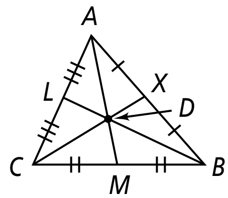


Name ______________________ Class ____________ Date ____________

Reteaching

Medians and Altitudes

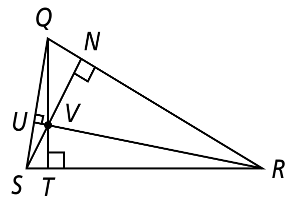
A *median* of a triangle is a segment that runs from one vertex of the triangle to the midpoint of the opposite side. The point of concurrency of the medians is called the *centroid.*



The medians of $\triangle ABC$ are $\overline{AM}$, $\overline{CX}$, and $\overline{BL}$.

The centroid is point *D*.

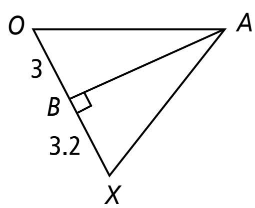
An *altitude* of a triangle is a segment that runs from one vertex perpendicular to the line that contains the opposite side. The *orthocenter* is the point of concurrency for the altitudes. An altitude may be inside or outside the triangle, or a side of the triangle.



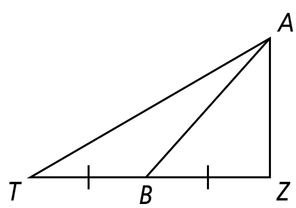
The altitudes of $\triangle QRS$ are $\overline{QT}$, $\overline{RU}$, and $\overline{SN}$.

The orthocenter is point *V*.

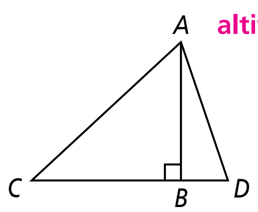
Determine whether $\overline{AB}$ is a *median*, an *altitude*, or *neither*.

1. 

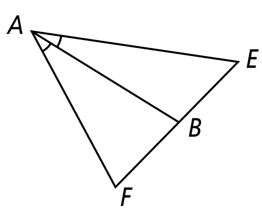
altitude

2. 

median

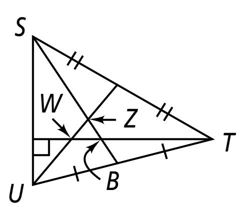
3. 

altitude

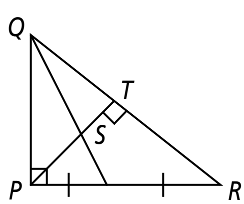
4. 

neither

5. Name the centroid. ***Z***



6. Name the orthocenter. ***P***



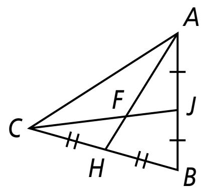
Name ______________________ Class ____________ Date ____________

Reteaching (continued)

Medians and Altitudes

The medians of a triangle intersect at a point two-thirds of the distance from a vertex to the opposite side. This is the Concurrency of Medians Theorem.

$\overline{CJ}$ and $\overline{AH}$ are medians of $\triangle ABC$ and point F is the centroid.



$$CF = \frac{2}{3}CJ$$

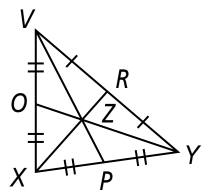
Problem

Point F is the centroid of $\triangle ABC$. If $CF = 30$, what is CJ?

$CF = \frac{2}{3}CJ$	Concurrency of Medians Theorem
$30 = \frac{2}{3} \times CJ$	Fill in known information.
$\frac{3}{2} \times 30 = CJ$	Multiply each side by $\frac{3}{2}$.
$45 = CJ$	Solve for CJ.

Exercises

In $\triangle VYX$, the centroid is Z. Use the diagram to solve the problems.



7. If $XR = 24$, find XZ and ZR. **16; 8**

8. If $XZ = 44$, find XR and ZR. **66; 22**

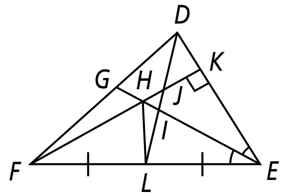
9. If $VZ = 14$, find VP and ZP. **21; 7**

10. If $VP = 51$, find VZ and ZP. **34; 17**

11. If $ZO = 10$, find YZ and YO. **20; 30**

12. If $YO = 18$, find YZ and ZO. **12; 6**

In Exercises 13–16, name each segment.



13. a median in $\triangle DEF$ **$\overline{DL}$**

14. an altitude in $\triangle DEF$ **$\overline{FK}$**

15. a median in $\triangle EHF$ **$\overline{HL}$**

16. an altitude in $\triangle HEK$ **$\overline{HK}$ or $\overline{KE}$**

Name ______________________ Class ____________ Date ____________

Reteaching

Indirect Proof

In an *indirect proof,* you prove a statement or conclusion to be true by proving the opposite of the statement to be false.

There are three steps to writing an indirect proof.

Step 1: State as a temporary assumption the opposite (negation) of what you want to prove.

Step 2: Show that this temporary assumption leads to a contradiction.

Step 3: Conclude that the temporary assumption is false and that what you want to prove must be true.

Problem

Given: There are 13 dogs in a show; some are long-haired and the rest are short-haired. There are more long-haired than short-haired dogs.

Prove: There are at least seven long-haired dogs in the show.

Step 1: Assume that fewer than seven long-haired dogs are in the show.

Step 2: Let ℓ be the number of long-haired dogs and s be the number of short-haired dogs. Because $\ell + s = 13$, $s = 13 - \ell$. If ℓ is less than 7, s is greater than or equal to 7. Therefore, s is greater than ℓ. This *contradicts* the statement that there are more long-haired than short-haired dogs.

Step 3: Therefore, there are at least seven long-haired dogs.

Exercises

Write the temporary assumption you would make as a first step in writing an indirect proof.

1. **Given:** an integer q; **Prove:** q is a factor of 34. **Assume q is not a factor of 34.**
2. **Given:** $\triangle XYZ$; **Prove:** $XY + XZ > YZ$. **Assume $XY + XZ \leq YZ$.**
3. **Given:** rectangle $GHIJ$; **Prove:** $m\angle G = 90$ **Assume $m\angle G \neq 90$.**
4. **Given:** $\overline{XY}$ and $\overline{XM}$; **Prove:** $XY = XM$ **Assume $XY \neq XM$.**

Write a statement that contradicts the given statement.

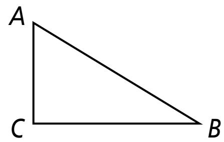
5. Whitney lives in an apartment. **Whitney does not live in an apartment.**
6. Marc does not have three sisters. **Marc has three sisters.**
7. $\angle 1$ is a right angle. **$\angle 1$ is an acute angle.**
8. Lines m and h intersect. **Lines m and h do not intersect.**

Name ______________________ Class __________ Date __________

Reteaching (continued)

Indirect Proof

Problem



Given: $\angle A$ and $\angle B$ are not complementary.
Prove: $\angle C$ is not a right angle.

Step 1: Assume that $\angle C$ is a right angle.

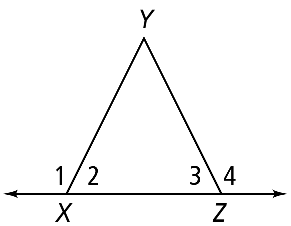
Step 2: If $\angle C$ is a right angle, then by the Triangle Angle-Sum Theorem, $m\angle A + m\angle B + 90 = 180$. So $m\angle A + m\angle B = 90$. Therefore, $\angle A$ and $\angle B$ are complementary. But $\angle A$ and $\angle B$ are not complementary.

Step 3: Therefore, $\angle C$ is not a right angle.

Exercises

Complete the proofs.

9. Arrange the statements given at the right to complete the steps of the indirect proof.



Given: $\overline{XY} \ncong \overline{YZ}$

Prove: $\angle 1 \ncong \angle 4$

Step 1: ? B

Step 2: ? D

Step 3: ? F

Step 4: ? E

Step 5: ? A

Step 6: ? C

A. But $\overline{XY} \ncong \overline{YZ}$.

B. Assume $\angle 1 \cong \angle 4$.

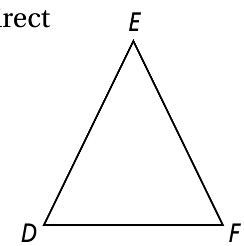
C. Therefore, $\angle 1 \ncong \angle 4$.

D. $\angle 1$ and $\angle 2$ are supplementary, and $\angle 3$ and $\angle 4$ are supplementary.

E. According to the Converse of the Isosceles Triangle Theorem, $XY = YZ$ or $\overline{XY} \cong \overline{YZ}$.

F. If $\angle 1 \cong \angle 4$, then by the Congruent Supplements Theorem, $\angle 2 \cong \angle 3$.

10. Complete the steps below to write a convincing argument using indirect reasoning.



Given: $\triangle DEF$ with $\angle D \ncong \angle F$

Prove: $\overline{EF} \ncong \overline{DE}$

Step 1: ? Assume $\overline{EF} \cong \overline{DE}$.

Step 2: ? If $\overline{EF} \cong \overline{DE}$, then by the Isosceles Triangle Theorem, $\angle D \cong \angle F$.

Step 3: ? But $\angle D \ncong \angle F$.

Step 4: ? Therefore, $\overline{EF} \ncong \overline{DE}$.

Name ______________________ Class ____________ Date ____________

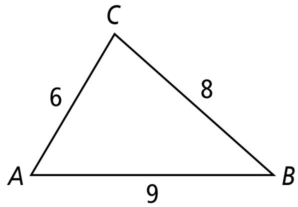
Reteaching

Inequalities in One Triangle

For any triangle, if two sides are not congruent, then the larger angle is opposite the longer side (Theorem 33). Conversely, if two angles are not congruent, then the longer side is opposite the larger angle (Theorem 34).

Problem

Use the triangle inequality theorems to answer the questions.

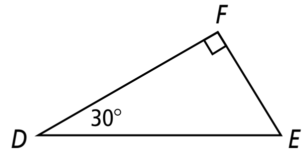


a. Which is the largest angle of $\triangle ABC$?

$\overline{AB}$ is the longest side of $\triangle ABC$. $\angle C$ lies opposite $\overline{AB}$.

$\angle C$ is the largest angle of $\triangle ABC$.

b. What is $m\angle E$? Which is the shortest side of $\triangle DEF$?



$m\angle D + m\angle E + m\angle F = 180$	Triangle Angle-Sum Theorem
$30 + m\angle E + 90 = 180$	Substitution
$120 + m\angle E = 180$	Addition
$m\angle E = 60$	Subtraction Property of Equality

$\angle D$ is the smallest angle of $\triangle DEF$. Because $\overline{FE}$ lies opposite $\angle D$, $\overline{FE}$ is the shortest side of $\triangle DEF$.

Exercises

1. Draw three triangles, one obtuse, one acute, and one right. Label the vertices. Exchange your triangles with a partner.

a. Identify the longest and shortest sides of each triangle.

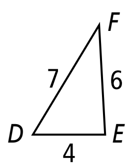
b. Identify the largest and smallest angles of each triangle.

c. Describe the relationship between the longest and shortest sides and the largest and smallest angles for each of your partner's triangles.

Check students' work. The longest side will be opposite the largest angle. The shortest side will be opposite the smallest angle.

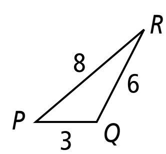
Which are the largest and smallest angles of each triangle?

2.



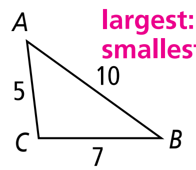
largest: $\angle DEF$; smallest: $\angle DFE$

3.



largest: $\angle PQR$; smallest: $\angle PRQ$

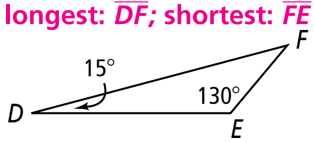
4.



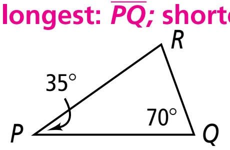
largest: $\angle ACB$; smallest: $\angle CBA$

Which are the longest and shortest sides of each triangle?

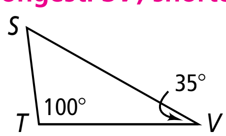
5. **longest: $\overline{DF}$; shortest: $\overline{FE}$**



6. **longest: $\overline{PQ}$; shortest: $\overline{RQ}$**



7. **longest: $\overline{SV}$; shortest: $\overline{ST}$**



Reteaching (continued)

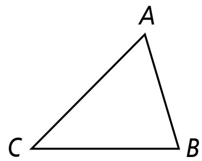
Inequalities in One Triangle

For any triangle, the sum of the lengths of any two sides is greater than the length of the third side. This is the *Triangle Inequality Theorem.*

$AB + BC > AC$

$AC + BC > AB$

$AB + AC > BC$



Problem

A. Can a triangle have side lengths 22, 33, and 25?

Compare the sum of two side lengths with the third side length.

$22 + 33 > 25$ $\quad$ $22 + 25 > 33$ $\quad$ $25 + 33 > 22$

A triangle *can* have these side lengths.

B. Can a triangle have side lengths 3, 7, and 11?

Compare the sum of two side lengths with the third side length.

$3 + 7 < 11$ $\quad$ $3 + 11 > 7$ $\quad$ $11 + 7 > 3$

A triangle *cannot* have these side lengths.

C. Two sides of a triangle are 11 and 12 ft long. What could be the length of the third side?

Set up inequalities using x to represent the length of the third side.

$x + 11 > 12$ $\quad$ $x + 12 > 11$ $\quad$ $11 + 12 > x$

$x > 1$ $\quad$ $x > -1$ $\quad$ $23 > x$

The side length can be any value between 1 and 23 ft long.

Exercises

8. Can a triangle have side lengths 2, 3, and 7? **no**

9. Can a triangle have side lengths 12, 13, and 7? **yes**

10. Can a triangle have side lengths 6, 8, and 9? **yes**

11. Two sides of a triangle are 5 cm and 3 cm. What could be the length of the third side? **less than 8 cm and greater than 2 cm**

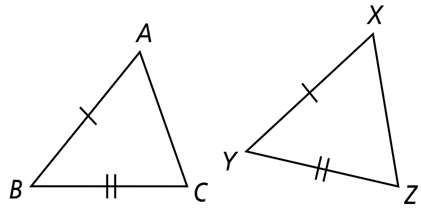
12. Two sides of a triangle are 15 ft and 12 ft. What could be the length of the third side? **less than 27 ft and greater than 3 ft**

Name ______________________ Class ____________ Date ____________

Reteaching

Inequalities in Two Triangles

Consider $\triangle ABC$ and $\triangle XYZ$. If $\overline{AB} \cong \overline{XY}$, $\overline{BC} \cong \overline{YZ}$, and $m\angle Y > m\angle B$, then $XZ > AC$. This is the *Hinge Theorem* (SAS Inequality Theorem).



Problem

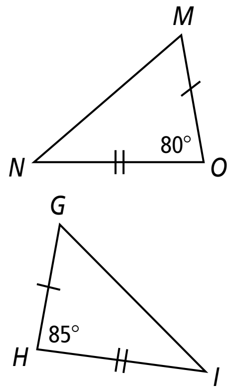
Which length is greater, *GI* or *MN*?

Identify congruent sides: $\overline{MO} \cong \overline{GH}$ and $\overline{NO} \cong \overline{HI}$.

Compare included angles: $m\angle H > m\angle O$.

By the Hinge Theorem, the side opposite the larger included angle is longer.

So, $GI > MN$.



Problem

At which time is the distance between the tip of a clock's hour hand and the tip of its minute hand greater, 3:00 or 3:10?

Think of the hour hand and the minute hand as two sides of a triangle whose lengths never change, and the distance between the tips of the hands as the third side. 3:00 and 3:10 can then be represented as triangles with two pairs of congruent sides. The distance between the tips of the hands is the side of the triangle opposite the included angle.

At 3:00, the measure of the angle formed by the hour hand and minute hand is 90°. At 3:10, the measure of the angle is less than 90°.

So, the distance between the tip of the hour hand and the tip of the minute hand is greater at 3:00.

Exercises

1. What is the inequality relationship between *LP* and *XA* in the figure at the right? ***XA > LP***

2. At which time is the distance between the tip of a clock's hour hand and the tip of its minute hand greater, 5:00 or 5:15? **5:00**