Name ______________________________ Class ____________ Date ____________

Reteaching

Congruent Figures

Given $ABCD \cong QRST$, find corresponding parts using the names. Order matters.

For example, $ABCD$ / $QRST$ — This shows that $\angle A$ corresponds to $\angle Q$. Therefore, $\angle A \cong \angle Q$.

For example, $ABCD$ / $QRST$ — This shows that $\overline{BC}$ corresponds to $\overline{RS}$. Therefore, $\overline{BC} \cong \overline{RS}$.

Exercises

Find corresponding parts using the order of the letters in the names.

1. Identify the remaining three pairs of corresponding angles and sides between $ABCD$ and $QRST$ using the circle technique shown above.

$\angle B \cong \angle R$, $\angle C \cong \angle S$, $\angle D \cong \angle T$, $\overline{AB} \cong \overline{QR}$, $\overline{CD} \cong \overline{ST}$, and $\overline{DA} \cong \overline{TQ}$

Angles: $ABCD$ $ABCD$ $ABCD$ Sides: $ABCD$ $ABCD$ $ABCD$

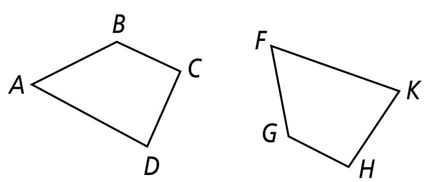
$QRST$ $QRST$ $QRST$ $QRST$ $QRST$ $QRST$

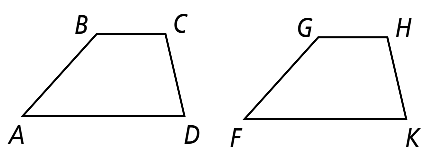
2. Which pair of corresponding sides is hardest to identify using this technique?

Answers may vary. Sample: $\overline{AD}$ and $\overline{QT}$

Find corresponding parts by redrawing figures.

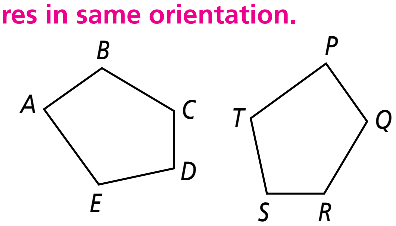
3. The two congruent figures below at the left have been redrawn at the right. Why are the corresponding parts easier to identify in the drawing at the right?





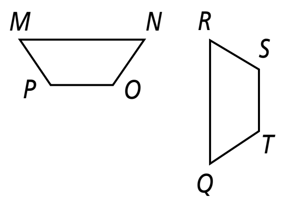
Answers may vary. Sample: The drawing at the right shows figures in same orientation.

4. Redraw the congruent polygons at the right in the same orientation. Identify all pairs of corresponding sides and angles. Check students' work. $\angle A$ and $\angle P$, $\angle B$ and $\angle Q$, $\angle C$ and $\angle R$, $\angle D$ and $\angle S$, $\angle E$ and $\angle T$, $\overline{AB}$ and $\overline{PQ}$, $\overline{BC}$ and $\overline{QR}$, $\overline{CD}$ and $\overline{RS}$, $\overline{DE}$ and $\overline{ST}$, and $\overline{EA}$ and $\overline{TP}$ all correspond.



5. $MNOP \cong QRST$. Identify all pairs of congruent sides and angles.

$\angle M \cong \angle Q$, $\angle N \cong \angle R$, $\angle O \cong \angle S$, $\angle P \cong \angle T$,
$\overline{MN} \cong \overline{QR}$, $\overline{NO} \cong \overline{RS}$, $\overline{OP} \cong \overline{ST}$, and $\overline{PM} \cong \overline{TQ}$



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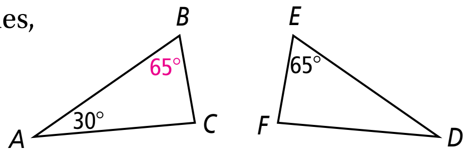
Reteaching (continued)

Congruent Figures

Problem

Given $\triangle ABC \cong \triangle DEF$, $m\angle A = 30$, and $m\angle E = 65$, what is $m\angle C$?

How might you solve this problem? Sketch both triangles, and put all the information on both diagrams.



$m\angle A = 30$; therefore, $m\angle D = 30$. How do you know? Because $\angle A$ and $\angle D$ are corresponding parts of congruent triangles.

Exercises

Work through the exercises below to solve the problem above.

6. What angle in $\triangle ABC$ has the same measure as $\angle E$? What is the measure of that angle? Add the information to your sketch of $\triangle ABC$.
$\angle B$; 65

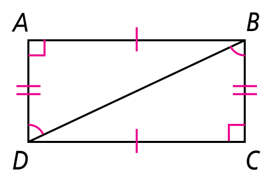
7. You know the measures of two angles in $\triangle ABC$. How can you find the measure of the third angle?
Answers may vary. Sample: Use Triangle Angle-Sum Thm. Set sum of all three angles equal to 180.

8. What is $m\angle C$? How did you find your answer?

85; answers may vary. Sample: $m\angle C = 180 - (60 + 35)$

Before writing a proof, add the information implied by each given statement to your sketch. Then use your sketch to help you with Exercises 9–12.

Add the information implied by each given statement.



9. Given: $\angle A$ and $\angle C$ are right angles.
$m\angle A = m\angle C = 90$, $\overline{DA} \perp \overline{AB}$ and $\overline{DC} \perp \overline{BC}$

10. Given: $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$.
ABCD is a parallelogram because it has opposite sides that are congruent.

11. Given: $\angle ADB \cong \angle CBD$.
$\overline{AD} \parallel \overline{BC}$

12. Can you conclude that $\angle ABD \cong \angle CDB$ using the given information above? If so, how?
Yes; use the Third Angles Thm.

13. How can you conclude that the third side of both triangles is congruent?
The third side is shared by both triangles and is congruent by the Refl. Prop. of Congruence.

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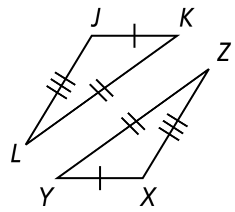
Reteaching

Triangle Congruence by SSS and SAS

You can prove that triangles are congruent using the two postulates below.

Postulate 4-1: Side-Side-Side (SSS) Postulate

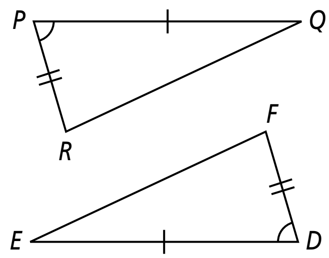


If all three sides of a triangle are congruent to all three sides of another triangle, then those two triangles are congruent.

If $\overline{JK} \cong \overline{XY}$, $\overline{KL} \cong \overline{YZ}$, and $\overline{JL} \cong \overline{XZ}$, then $\triangle JKL \cong \triangle XYZ$.

In a triangle, the angle formed by any two sides is called the *included angle* for those sides.

Postulate 4-2: Side-Angle-Side (SAS) Postulate

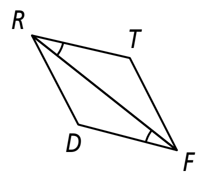


If two sides and the included angle of a triangle are congruent to two sides and the included angle of another triangle, then those two triangles are congruent.

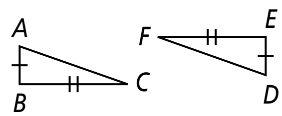
If $\overline{PQ} \cong \overline{DE}$, $\overline{PR} \cong \overline{DF}$, and $\angle P \cong \angle D$, then $\triangle PQR \cong \triangle DEF$.

$\angle P$ is included by $\overline{QP}$ and $\overline{PR}$. $\angle D$ is included by $\overline{ED}$ and $\overline{DF}$.

Exercises



1. What other information do you need to prove $\triangle TRF \cong \triangle DFR$ by SAS? Explain. **$\overline{DF} \cong \overline{TR}$; by the Reflexive Property of Congruence, $\overline{RF} \cong \overline{FR}$. It is given that $\angle TRF \cong \angle DFR$. These are the included angles for the corresponding congruent sides.**

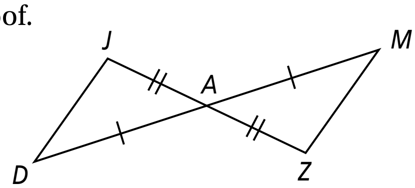


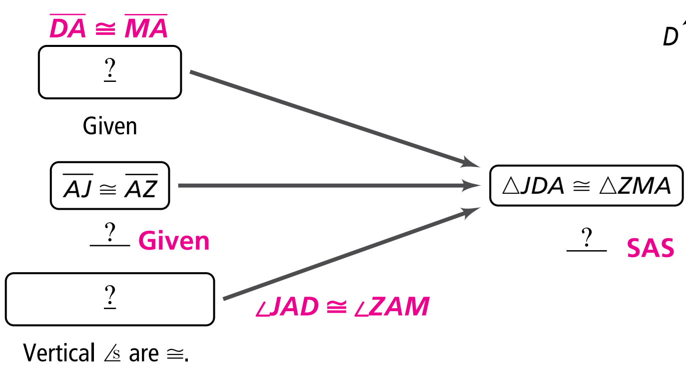
2. What other information do you need to prove $\triangle ABC \cong \triangle DEF$ by SAS? Explain.
$\angle B \cong \angle E$; These are the included angles between the corresponding congruent sides.

3. Developing Proof Copy and complete the flow proof.

Given: $\overline{DA} \cong \overline{MA}$, $\overline{AJ} \cong \overline{AZ}$

Prove: $\triangle JDA \cong \triangle ZMA$



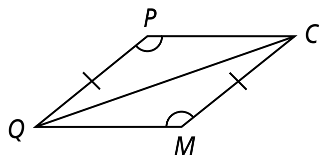


Name ______________________ Class ____________ Date ____________

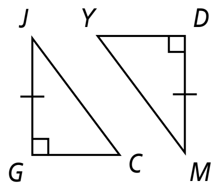
Reteaching (continued)

Triangle Congruence by SSS and SAS

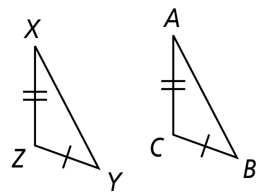
Would you use SSS or SAS to prove the triangles congruent? If there is not enough information to prove the triangles congruent by SSS or SAS, write *not enough information.* Explain your answer.

4. 

Not enough information; two pairs of corresponding sides are congruent, but the congruent angles are not the included angles.

5. 

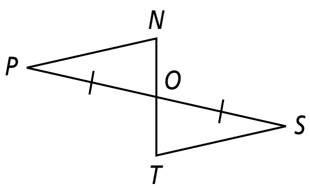
Not enough information; you need to know if $\overline{GC} \cong \overline{DY}$.

6. 

Not enough information; only two pairs of corresponding sides are congruent. You need to know if $\overline{AB} \cong \overline{XY}$ or $\angle Z \cong \angle C$.

7. Given: $\overline{PO} \cong \overline{SO}$, O is the midpoint of $\overline{NT}$.

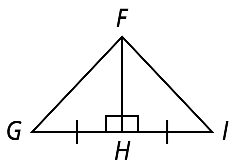
Prove: $\triangle NOP \cong \triangle TOS$



Statements: 1) $\overline{PO} \cong \overline{SO}$; 2) O is the midpoint of $\overline{NT}$; 3) $\overline{NO} \cong \overline{TO}$; 4) $\angle NOP \cong \angle TOS$; 5) $\triangle NOP \cong \triangle TOS$; Reasons: 1) Given; 2) Given; 3) Def. of midpoint; 4) Vert. $\angle$s are $\cong$; 5) SAS

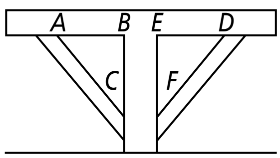
8. Given: $\overline{HI} \cong \overline{HG}$, $\overline{FH} \perp \overline{GI}$

Prove: $\triangle FHI \cong \triangle FHG$



Statements: 1) $\overline{FH} \cong \overline{FH}$; 2) $\overline{HI} \cong \overline{HG}$, $\overline{FH} \perp \overline{GI}$; 3) $\angle FHG$ and $\angle FHI$ are rt. $\angle$s; 4) $\angle FHG \cong \angle FHI$; 5) $\triangle FHI \cong \triangle FHG$; Reasons: 1) Refl. Prop.; 2) Given; 3) Def. of perpendicular; 4) All rt. $\angle$s are $\cong$; 5) SAS

9. A carpenter is building a support for a bird feeder. He wants the triangles on either side of the vertical post to be congruent. He measures and finds that $\overline{AB} \cong \overline{DE}$ and that $\overline{AC} \cong \overline{DF}$. What would he need to measure to prove that the triangles are congruent using SAS? What would he need to measure to prove that they are congruent using SSS?



For SAS, he would need to determine if $\angle BAC \cong \angle EDF$; for SSS, he would need to determine if $\overline{BC} \cong \overline{EF}$.

10. An artist is drawing two triangles. She draws each so that two sides are 4 in. and 5 in. long and an angle is 55°. Are her triangles congruent? Explain.

Answers may vary. Sample: Maybe; if both the 55° angles are between the 4-in. and 5-in. sides, then the triangles are congruent by SAS.

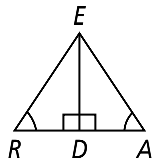
Name ______________________ Class ____________ Date ____________

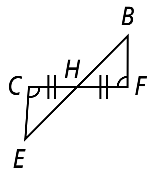
Reteaching

Triangle Congruence by ASA and AAS

Problem

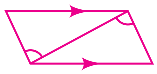
Can the ASA Postulate or the AAS Theorem be applied directly to prove the triangles congruent?

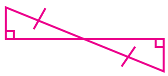




a. Because ∠*RDE* and ∠*ADE* are right angles, they are congruent. $\overline{ED} \cong \overline{ED}$ by the Reflexive Property of ≅, and it is given that ∠*R* ≅ ∠*A*. Therefore, △*RDE* ≅ △*ADE* by the AAS Theorem.

b. It is given that $\overline{CH} \cong \overline{FH}$ and ∠*F* ≅ ∠*C*. Because ∠*CHE* and ∠*FHB* are vertical angles, they are congruent. Therefore, △*CHE* ≅ △*FHB* by the ASA Postulate.

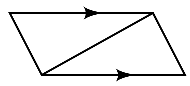
1. 

2. 

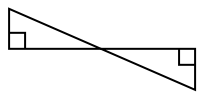
Exercises

Indicate congruences.

1. Copy the top figure at the right. Mark the figure with the angle congruence and side congruence symbols that you would need to prove the triangles congruent by the ASA Postulate.



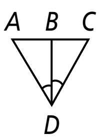
2. Copy the second figure shown. Mark the figure with the angle congruence and side congruence symbols that you would need to prove the triangles congruent by the AAS Theorem.



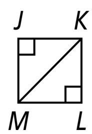
3. Draw and mark two triangles that are congruent by either the ASA Postulate or the AAS Theorem. **Check students' work.**

What additional information would you need to prove each pair of triangles congruent by the stated postulate or theorem?

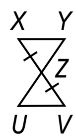
4. ASA Postulate
∠ABD ≅ ∠CBD



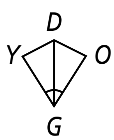
5. AAS Theorem
∠JMK ≅ ∠LKM, ∠JKM ≅ ∠LMK, ∠JMK ≅ ∠LMK, or ∠JKM ≅ ∠LKM



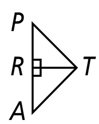
6. ASA Postulate
∠ZXY ≅ ∠ZVU



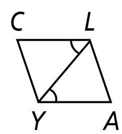
7. AAS Theorem
∠Y ≅ ∠O



8. AAS Theorem
∠P ≅ ∠A



9. ASA Postulate
∠CYL ≅ ∠ALY



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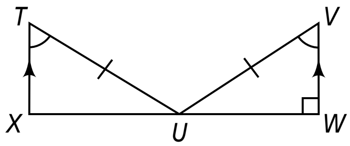
Reteaching (continued)

Triangle Congruence by ASA and AAS

10. Provide the reason for each step in the two-column proof.

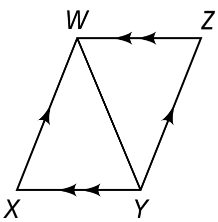


Given: $\overline{TX} \parallel \overline{VW}$, $\overline{TU} \cong \overline{VU}$, $\angle XTU \cong \angle WVU$,
$\angle UWV$ is a right angle.

Prove: $\triangle TUX \cong \triangle VUW$

Statements	Reasons
1) $\angle UWV$ is a right angle.	1) ___?___ **Given**
2) $\overline{VW} \perp \overline{XW}$	2) ___?___ **Definition of perpendicular lines**
3) $\overline{TX} \parallel \overline{VW}$	3) ___?___ **Given**
4) $\overline{TX} \perp \overline{XW}$	4) ___?___ **Perpendicular Transversal Theorem**
5) $\angle UXT$ is a right angle.	5) ___?___ **Definition of perpendicular lines**
6) $\angle UWV \cong \angle UXT$	6) ___?___ **All right angles are congruent.**
7) $\overline{TU} \cong \overline{VU}$	7) ___?___ **Given**
8) $\angle XTU \cong \angle WVU$	8) ___?___ **Given**
9) $\triangle TUX \cong \triangle VUW$	9) ___?___ **AAS Theorem**

11. Write a paragraph proof.

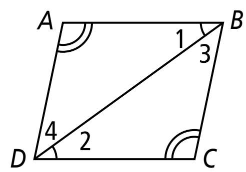


Given: $\overline{WX} \parallel \overline{ZY}$; $\overline{WZ} \parallel \overline{XY}$

Prove: $\triangle WXY \cong \triangle YZW$

It is given that $\overline{WX} \parallel \overline{ZY}$ and $\overline{WZ} \parallel \overline{XY}$, so $\angle XWY \cong \angle ZYW$ and $\angle XYW \cong \angle ZWY$, by the Alternate Interior ∠s Thm. $\overline{WY} \cong \overline{YW}$ by the Reflexive Property of ≅. So, by ASA Post. $\triangle WXY \cong \triangle YZW$.

12. Developing Proof Complete the proof by filling in the blanks.



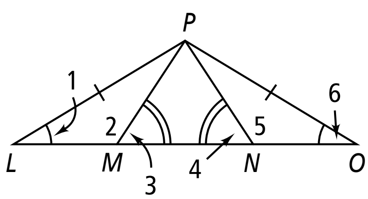
Given: $\angle A \cong \angle C$, $\angle 1 \cong \angle 2$

Prove: $\triangle ABD \cong \triangle CDB$

Proof: $\angle A \cong \angle C$ and $\angle 1 \cong \angle 2$ are given. $\overline{DB} \cong \overline{BD}$ by ___?___. **Refl. Prop. of Congruence**

So, $\triangle ABD \cong \triangle CDB$ by ___?___. **AAS**

13. Write a paragraph proof.



Given: $\angle 1 \cong \angle 6$, $\angle 3 \cong \angle 4$, $\overline{LP} \cong \overline{OP}$

Prove: $\triangle LMP \cong \triangle ONP$

$\angle 3 \cong \angle 4$ is given. Therefore, $m\angle 3 = m\angle 4$, by def. of ≅ ∠s. Because $\angle 2$ and $\angle 3$ are linear pairs, and $\angle 4$ and $\angle 5$ are linear pairs, the pairs of angles are suppl. Therefore, $\angle 2 \cong \angle 5$ by the Congruent Suppl. Thm. $\angle 1 \cong \angle 6$ and $\overline{LP} \cong \overline{OP}$ are given, so $\triangle LMP \cong \triangle ONP$, by the AAS Thm.

Name ______________________ Class ____________ Date ____________

Reteaching

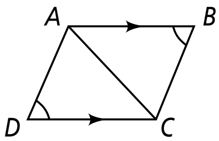
Using Corresponding Parts of Congruent Triangles

If you can show that two triangles are congruent, then you can show that all the corresponding angles and sides of the triangles are congruent.

Problem

Given: $\overline{AB} \parallel \overline{DC}$, $\angle B \cong \angle D$

Prove: $\overline{BC} \cong \overline{DA}$



In this case you know that $\overline{AB} \parallel \overline{DC}$. $\overline{AC}$ forms a transversal and creates a pair of alternate interior angles, $\angle BAC$ and $\angle DCA$.

You have two pairs of congruent angles, $\angle BAC \cong \angle DCA$ and $\angle B \cong \angle D$. Because you know that the shared side is congruent to itself, you can use AAS to show that the triangles are congruent. Then use the fact that corresponding parts are congruent to show that $\overline{BC} \cong \overline{DA}$. Here is the proof:

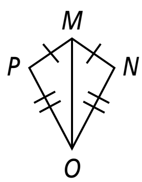
Statements	Reasons
1) $\overline{AB} \parallel \overline{DC}$	1) Given
2) $\angle BAC \cong \angle DCA$	2) Alternate Interior Angles Theorem
3) $\angle B \cong \angle D$	3) Given
4) $\overline{AC} \cong \overline{CA}$	4) Reflexive Property of Congruence
5) $\triangle ABC \cong \triangle CDA$	5) AAS
6) $\overline{BC} \cong \overline{DA}$	6) CPCTC

Exercises

1. Write a two-column proof.

Given: $\overline{MN} \cong \overline{MP}$, $\overline{NO} \cong \overline{PO}$

Prove: $\angle N \cong \angle P$



Statements	Reasons
1) ___?___ $\overline{MN} \cong \overline{MP}$, $\overline{NO} \cong \overline{PO}$	1) Given
2) $\overline{MO} \cong \overline{MO}$	2) ___?___ Reflexive Property of $\cong$
3) ___?___ $\triangle MNO \cong \triangle MPO$	3) ___?___ SSS
4) $\angle N \cong \angle P$	4) ___?___ CPCTC

Name ______________________________ Class ____________ Date ____________

Reteaching (continued)

Using Corresponding Parts of Congruent Triangles

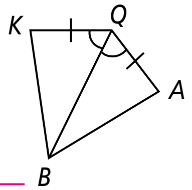
2. Write a two-column proof.

Given: $\overline{PT}$ is a median and an altitude of $\triangle PRS$.

Prove: $\overline{PT}$ bisects $\angle RPS$.

Statements	Reasons
1) $\overline{PT}$ is a median of $\triangle PRS$.	1) __?__ **Given**
2) __?__ **T is the midpoint of $\overline{RS}$.**	2) Definition of median
3) __?__ **$\overline{RT} \cong \overline{ST}$**	3) Definition of midpoint
4) $\overline{PT}$ is an altitude of $\triangle PRS$.	4) __?__ **Given**
5) $\overline{PT} \perp \overline{RS}$	5) __?__ **Definition of altitude**
6) $\angle PTS$ and $\angle PTR$ are right angles.	6) __?__ **Definition of perpendicular**
7) __?__ **$\angle PTS \cong \angle PTR$**	7) All right angles are congruent.
8) __?__ **$\overline{PT} \cong \overline{PT}$**	8) Reflexive Property of Congruence
9) __?__ **$\triangle PTS \cong \triangle PTR$**	9) SAS
10) $\angle TPS \cong \angle TPR$	10) __?__ **CPCTC**
11) __?__ **$\overline{PT}$ bisects $\angle RPS$.**	11) __?__ **Definition of angle bisector**

3. Write a two-column proof.

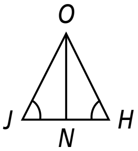


Given: $\overline{QK} \cong \overline{QA}$; $\overline{QB}$ bisects $\angle KQA$.

Prove: $\overline{KB} \cong \overline{AB}$

Statements	Reasons
1) $\overline{QK} \cong \overline{QA}$; $\overline{QB}$ bisects $\angle KQA$.	**1) Given**
2) $\angle KQB \cong \angle AQB$	**2) Def. of $\angle$ bis.**
3) $\overline{BQ} \cong \overline{BQ}$	**3) Refl. Prop. of Congruence**
4) $\triangle KBQ \cong \triangle ABQ$	**4) SAS**
5) $\overline{KB} \cong \overline{AB}$	**5) CPCTC**

4. Write a two-column proof.



Given: $\overline{ON}$ bisects $\angle JOH$, $\angle J \cong \angle H$

Prove: $\overline{JN} \cong \overline{HN}$

Statements	Reasons
1) $\overline{ON}$ bisects $\angle JOH$, $\angle J \cong \angle H$	**1) Given**
2) $\angle JON \cong \angle HON$	**2) Def. of $\angle$ bis.**
3) $\overline{ON} \cong \overline{ON}$	**3) Refl. Prop. of Congruence**
4) $\triangle JON \cong \triangle HON$	**4) AAS**
5) $\overline{JN} \cong \overline{HN}$	**5) CPCTC**

Name ______________________ Class ____________ Date ____________

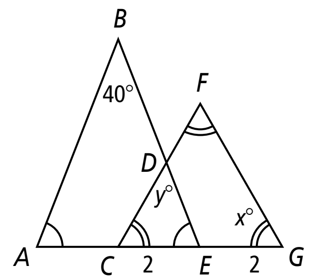
Reteaching (continued)

Isosceles and Equilateral Triangles

Problem

What is the value of x?

Because x is the measure of an angle in an equilateral triangle, $x = 60$.



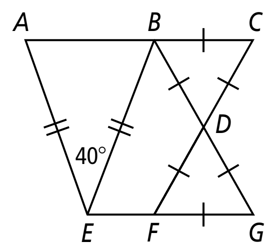
Problem

What is the value of y?

$m\angle DCE + m\angle DEC + m\angle EDC = 180$	There are 180° in a triangle.
$60 + 70 + y = 180$	Substitution Property
$y = 50$	Subtraction Property of Equality

Exercises

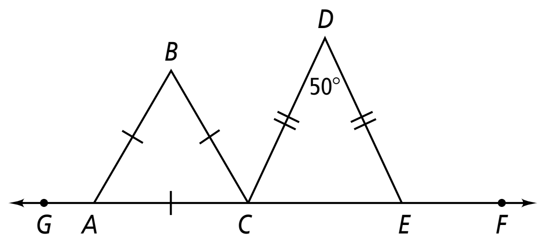
Complete each statement. Explain why it is true.



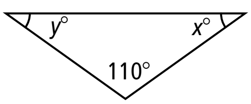
1. $\angle EAB \cong$ __?__
   $\angle EBA$; base angles of an isosceles triangle are congruent.
2. $\angle BCD \cong$ __?__ $\cong \angle DBC$
   $\angle CDB$; the angles of an equilateral triangle are congruent.
3. $\overline{FG} \cong$ __?__ $\cong \overline{DF}$
   $\overline{GD}$; the sides of an equilateral triangle are congruent.

Determine the measure of the indicated angle.

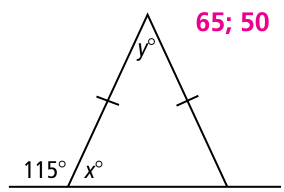
4. $\angle ACB$ **60**
5. $\angle DCE$ **65**
6. $\angle BCD$ **55**



Algebra **Find the value of x and y.**

7. 

35; 35

8. 

65; 50

9. **Reasoning** An exterior angle of an isosceles triangle has a measure 140. Find two possible sets of measures for the angles of the triangle. **40, 40, 100; 40, 70, 70**

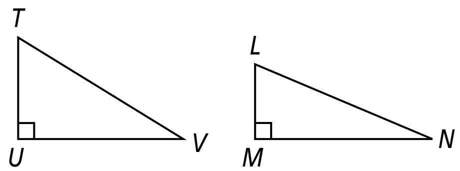
Name ______________________ Class ____________ Date ____________

Reteaching (continued)

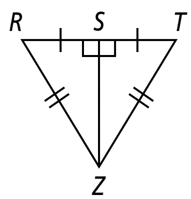
Congruence in Right Triangles

Exercises

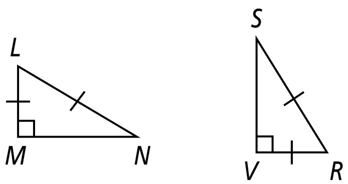
Determine if the given triangles are congruent by the Hypotenuse-Leg Theorem. If so, write the triangle congruence statement.

1. 

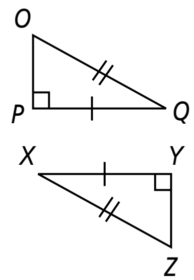
not congruent

2. 

$\triangle RSZ \cong \triangle TSZ$

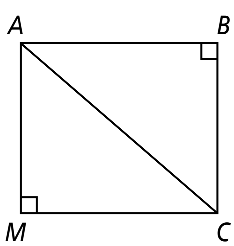
3. 

$\triangle LMN \cong \triangle RVS$

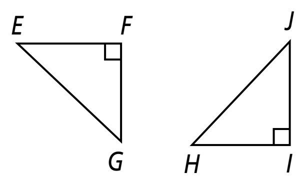
4. 

$\triangle OPQ \cong \triangle ZYX$

Measure the hypotenuse and length of the legs of the given triangles with a ruler to determine if the triangles are congruent. If so, write the triangle congruence statement.

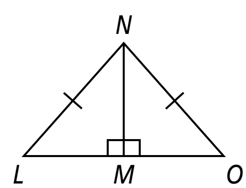
5. 

$\triangle ABC \cong \triangle CMA$

6. 

$\triangle EFG \cong \triangle HIJ$

7. Explain why $\triangle LMN \cong \triangle OMN$. Use the Hypotenuse-Leg Theorem.



Because $\angle NML$ and $\angle NMO$ are right angles, both triangles are right triangles. It is given that their hypotenuses are congruent. Because they share a leg, one pair of corresponding legs is congruent. All criteria are met for the triangles to be congruent by the Hypotenuse-Leg Theorem.

8. Visualize $\triangle ABC$ and $\triangle DEF$, where $AB = EF$ and $CA = FD$. What else must be true about these two triangles to prove that the triangles are congruent using the Hypotenuse-Leg Theorem? Write a congruence statement.

$\angle B$ and $\angle E$ are right angles, or $\angle C$ and $\angle D$ are right angles. $\triangle ABC \cong \triangle DEF$ or $\triangle ABC \cong \triangle FED$.

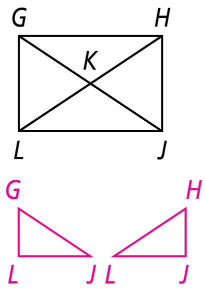
Name ______________________________ Class ____________ Date ____________

Reteaching (continued)

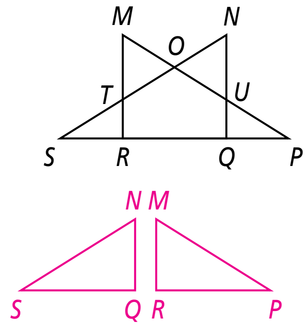
Congruence in Overlapping Triangles

Separate and redraw the overlapping triangles. Identify the vertices.

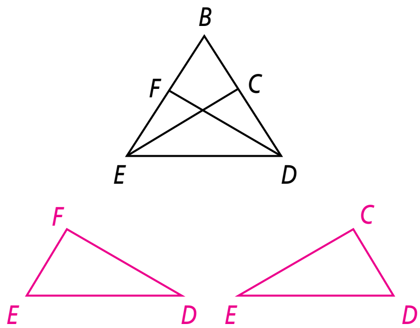
1. $\triangle GLJ$ and $\triangle HJL$



2. $\triangle MRP$ and $\triangle NQS$



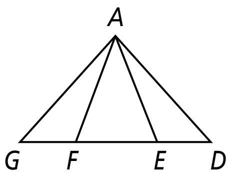
3. $\triangle FED$ and $\triangle CDE$



Fill in the blanks for the two-column proof.

4. Given: $\angle AEG \cong \angle AFD$, $\overline{AE} \cong \overline{AF}$, $\overline{GE} \cong \overline{FD}$

Prove: $\triangle AFG \cong \triangle AED$

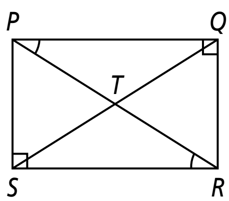


Statements	Reasons
1) $\angle AEG \cong \angle AFD$, $\overline{AE} \cong \overline{AF}$, $\overline{GE} \cong \overline{FD}$	1) ? **Given**
2) ? **$\triangle AEG \cong \triangle AFD$**	2) SAS
3) $\overline{AG} \cong \overline{AD}$, $\angle G \cong \angle D$	3) ? **CPCTC**
4) ? **$\overline{GE} \cong \overline{FD}$**	4) Given
5) $GE = FD$	5) ? **Def. of $\cong$**
6) $GF + FE = GE$, $FE + ED = FD$	6) ? **Seg. Addition Post.**
7) $GF + FE = FE + ED$	7) ? **Substitution Property**
8) ? **$GF = ED$**	8) Subtr. Prop. of Equality
9) ? **$\triangle AFG \cong \triangle AED$**	9) ? **SAS**

Use the plan to write a two-column proof.

5. Given: $\angle PSR$ and $\angle PQR$ are right angles, $\angle QPR \cong \angle SRP$.

Prove: $\triangle STR \cong \triangle QTP$



Plan for Proof:

Prove $\triangle QPR \cong \triangle SRP$ by AAS. Then use CPCTC and vertical angles to prove $\triangle STR \cong \triangle QTP$ by AAS.

Statements: 1) $\angle PSR$ and $\angle PQR$ are rt. ∠s; $\angle QPR \cong \angle SRP$; 2) $\angle PSR \cong \angle RQP$; 3) $\overline{PR} \cong \overline{RP}$; 4) $\triangle QPR \cong \triangle SRP$; 5) $\angle STR \cong \angle QTP$; 6) $\overline{PQ} \cong \overline{RS}$; 7) $\triangle STR \cong \triangle QTP$; Reasons: 1) Given; 2) Rt. ∠s are congruent; 3) Refl. Prop. of $\cong$; 4) AAS; 5) Vert. ∠s are $\cong$; 6) CPCTC; 7) AAS

Name ______________________ Class __________ Date __________

Reteaching

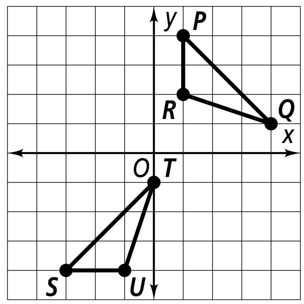
Congruence Transformations

Because rigid motions preserve distance and angle measure, the image of a rigid motion or composition of rigid motions is congruent to the preimage. Congruence can be defined by rigid motions as follows.

Two figures are *congruent* if and only if there is a sequence of one or more rigid motions that map one figure onto the other.

Because rigid motions map figures to congruent figures, rigid motions and compositions of rigid motions are also called *congruence transformations*. If two figures are congruent, you can find a congruence transformation that maps one figure to the other.

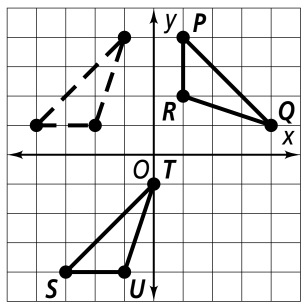
Problem



In the figure at the right, $\triangle PQR \cong \triangle STU$. What is a congruence transformation that maps $\triangle PQR$ to $\triangle STU$?

$\triangle STU$ appears to have the same shape and orientation as $\triangle PQR$, but rotated 90°, so start by applying the rotation $r_{(90°, O)}$ on the vertices of $\triangle PQR$.

$r_{(90°, O)}(P) = (-4, 1)$, $r_{(90°, O)}(Q) = (-1, 4)$, $r_{(90°, O)}(r) = (-2, 1)$

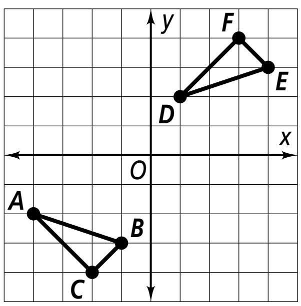


Graph the image $r_{(90°, O)}(\triangle PQR)$. A translation of 1 unit to the right and 5 units down maps the image to $\triangle STU$.

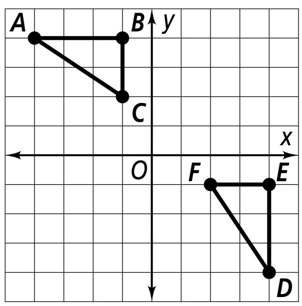
Therefore, $(T_{<1, -5>} \circ r_{(90°, O)})(\triangle PQR) = \triangle STU$.

Exercises

Find a congruence transformation that maps $\triangle ABC$ to $\triangle DEF$.

1. 

Answers may vary. Sample: $(R_{x\text{-axis}} \circ T_{<5, 0>})(\triangle ABC) = \triangle DEF$

2. 

Answers may vary. Sample: $(r_{(270°, O)} \circ R_{y\text{-axis}})(\triangle ABC) = \triangle DEF$

Name ______________________ Class ____________ Date ____________

Reteaching (continued)

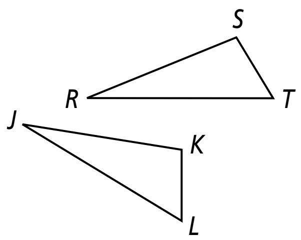
Congruence Transformations

If you can show that a congruence transformation exists from one figure to another, then you have shown that the figures are congruent.

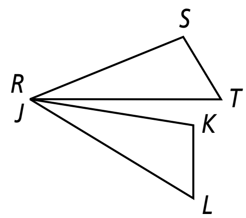
Problem

Verify the SSS Postulate by using a congruence transformation.

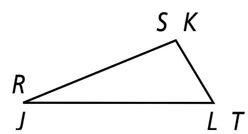
Given: $\overline{JK} \cong \overline{RS}$, $\overline{KL} \cong \overline{ST}$, $\overline{LJ} \cong \overline{TR}$
Prove: $\triangle JKL \cong \triangle RST$



Start by translating $\triangle JKL$ so that points J and R coincide.



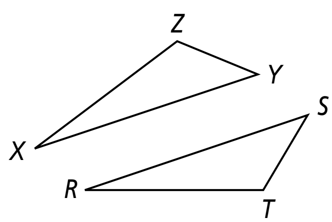
Because you are given that $\overline{JK} \cong \overline{RS}$, there is a rigid motion that maps $\overline{JK}$ onto $\overline{RS}$ by rotating $\triangle JKL$ about point R so that $\overline{JK}$ and $\overline{RS}$ coincide. Thus, there is a congruence transformation that maps $\triangle JKL$ to $\triangle RST$, so $\triangle JKL \cong \triangle RST$.



Exercises

3. Verify the SAS Postulate for triangle congruence by using congruence transformations.

   Given: $\angle R \cong \angle X$, $\overline{RS} \cong \overline{XY}$, $\overline{ST} \cong \overline{YZ}$
   Prove: $\triangle RST \cong \triangle XYZ$
   Answers may vary. Sample: Since $\overline{RS} \cong \overline{XY}$, translate $\triangle RST$ so $\overline{RS}$ coincides with $\overline{XY}$. Then reflect $\triangle RST$ across $\overline{XY}$ to complete the transformation.



4. Verify the ASA Postulate for triangle congruence by using congruence transformations.

   Given: $\angle A \cong \angle J$, $\angle B \cong \angle K$, $\overline{AB} \cong \overline{JK}$
   Prove: $\triangle ABC \cong \triangle JKL$
   Answers may vary. Sample: Translate $\triangle ABC$ so that points C and L coincide. Then rotate $\triangle ABC$ about point L until $\overline{AB}$ and $\overline{JK}$ coincide.