Name ________________________ Class ____________ Date ____________

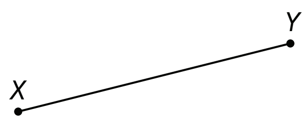
Reteaching

Basic Constructions

Before you start the construction, THINK about the goal. Then SKETCH the segment or angle you are trying to construct. Next EXPLAIN the purpose of each step in the construction as you complete it.

Problem

Construct $\overline{AB}$ so that $\overline{AB}$ is congruent to $\overline{XY}$.



THINK: Can you describe what the problem is asking for in your own words?
You want to draw a line segment that has the same length as one you are given.

SKETCH: Sketch a segment and label it as $\overline{AB}$. Why do you start with a sketch?
A sketch helps you to see what you need to construct.

EXPLAIN: First, draw a ray. What is the purpose of drawing a ray?
The ray is a part of a line on which you can mark off the correct length of $\overline{AB}$.

Second, measure the length of $\overline{XY}$, using the compass. What is the purpose of measuring the length of $\overline{XY}$?
You need this measure to mark the same length on the ray.

Finally, mark the length of $\overline{XY}$ on the ray, using the compass. Why do you mark the length of $\overline{XY}$ on your ray?
This completes your construction of $\overline{AB}$, *which is congruent to* $\overline{XY}$.

Exercises

Analyze the construction of a congruent angle and bisectors.

1. Analyze the construction of a perpendicular bisector. First draw $\overline{XY}$.

a. THINK about what it means to construct a perpendicular bisector. What is your goal? **Answers may vary. Sample: to draw a segment that is perpendicular to $\overline{XY}$ and that divides it into two equal pieces**

b. SKETCH a perpendicular bisector to $\overline{XY}$. **Check students' work.**

c. EXPLAIN the first two steps. What is the purpose of drawing an arc from each endpoint? **Answers may vary. Sample: to form two intersections to become two points of the perpendicular bisector**

d. EXPLAIN the last step. What is the purpose of drawing the segment between the intersections of the arcs?
Answers may vary. Sample: to draw the perpendicular bisector

Reteaching (continued)

Basic Constructions

2. Analyze the construction of a congruent angle. First draw $\angle X$.

a. THINK about what it means to construct a congruent angle. What is your goal?
Answers may vary. Sample: to draw an angle of the same size as the given angle

b. SKETCH $\angle Y$ congruent to $\angle X$. Your goal is to construct an angle the same size as $\angle Y$. **Check students' work.**

c. EXPLAIN the first step (drawing a ray). What is the purpose of the first step?
Answers may vary. Sample: to make a ray to serve as one side of the new congruent angle

d. EXPLAIN the second step (drawing an arc). What is the purpose of the second step? **Answers may vary. Sample: to measure the opening of the given angle**

e. EXPLAIN the third step (drawing an arc). What is the purpose of the third step? **Answers may vary. Sample: to mark the size of the given angle on the ray**

f. EXPLAIN the fourth step (drawing an arc). What is the purpose of the fourth step? **Answers may vary. Sample: to find the point of intersection so you can draw the other side of the angle**

g. EXPLAIN the fifth step (drawing a segment). What is the purpose of the fifth step? **Answers may vary. Sample: to draw the congruent angle**

3. Analyze the construction of an angle bisector. First draw $\angle W$.

a. THINK about what it means to construct an angle bisector. What is your goal? **Answers may vary. Sample: to divide a given angle into two equal angles**

b. SKETCH a ray that makes $\angle V$ congruent to $\frac{1}{2}\angle W$, where $\angle V$ shares a ray with $\angle W$ and has its other ray inside $\angle W$. Your goal is to construct a ray that bisects an angle. **Check students' work.**

c. EXPLAIN the first step (drawing an arc). What is the purpose of the first step? **Answers may vary. Sample: to locate two points of intersection to use to draw the other endpoint of the angle bisector**

d. EXPLAIN the second step (drawing two arcs). What is the purpose of the second step? **Answers may vary. Sample: to locate the point of intersection that will be another point on the angle bisector**

e. EXPLAIN the third step (drawing a ray). What is the purpose of the third step? **Answers may vary. Sample: to divide the given angle into two equal smaller angles**

Name ______________________ Class ____________ Date ____________

Reteaching

Patterns and Inductive Reasoning

Inductive reasoning is a type of reasoning in which you look at a pattern and then make some type of prediction based on the pattern. These predictions are also called conjectures. A *conjecture* is a statement about what you think will happen based on the pattern you observed.

Problem

Which conjectures below are reasonable? Which are not?

Pattern	Conjecture
Pattern 1: Every day for the two weeks that Alba visited Cairo, the weather was hot and dry.	Alba thought to herself, "The weather is always hot and dry in this city."
Pattern 2: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, . . .	Each number increases by 5. The next two numbers are most likely 55 and 60.
Pattern 3: Dani and Liz both examined this pattern of letters: A, BB, CCC, . . .	Dani was sure that the next two terms of the pattern would be DDDD and EEEEE. Liz wasn't so sure. She thought the pattern might repeat and the next two elements would be A and BB.

Pattern 1: The conjecture for Pattern 1 is probably not correct. In most cities, the weather will not be hot and dry all the time.

Pattern 2: You are given enough numbers in the pattern to assume that the numbers continue to increase by five. The conjecture is probably correct.

Pattern 3: Only three terms of the pattern are shown. This makes it difficult to determine what rule the pattern follows. Either Dani or Liz could be correct, or they could both be incorrect.

Remember that conjectures are never the final goal in a complete reasoning process. They are simply the first step to figuring out a problem.

Exercises

Make a conjecture about the rule these patterns follow.

1. 3, 6, 9, 12, 15, . . .
The numbers increase by 3.

2. 9, 3, 1, $\frac{1}{3}$, $\frac{1}{9}$, . . .
The numbers are divided by 3.

3. A, C, E, G, I, K, M, . . .
One letter of the alphabet is skipped between each term.

4. 0, 5, -2, 3, -4, 1, -6, . . .
Add 5, then subtract 7.

5. 4, 8, 16, 32, 64, 128, . . .
The numbers are multiplied by 2.

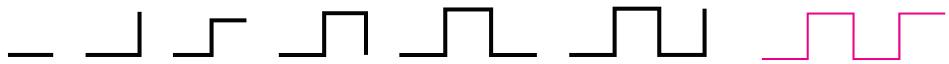
6. 0.1, 0.01, 0.001, 0.0001, . . .
The numbers are divided by 10.

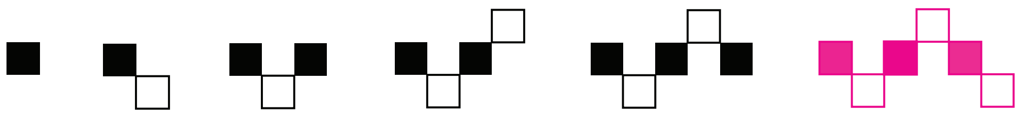
Name ______________________ Class __________ Date __________

Reteaching (continued)

Patterns and Inductive Reasoning

Draw the next figure in each sequence.

7. 

8. 

One way to show that a conjecture is not true is to find a *counterexample.* A *counterexample* is an instance in which the conjectured pattern does not work. Only one counterexample is needed to prove a conjecture false. For example, one rainy or cool day in Cairo would prove to Alba that it is not always hot and dry there.

Exercises

Find one counterexample to show that each conjecture is false.

9. All vehicles on the highway have exactly four wheels.
Sample: Motorcycles have two wheels.

10. All states in the United States share a border with another state.
Sample: Hawaii

11. All plurals end with the letter *s.*
Sample: geese

12. The difference between two integers is always positive.
Sample: $-3 - 2 = -5$

13. All pentagons have exactly five congruent sides.
Sample: an irregular pentagon

14. All numbers that are divisible by 3 are also divisible by 6.
Sample: 15

15. All whole numbers are greater than their opposites.
0

16. All prime numbers are odd integers.
2

Name ______________________________ Class ____________ Date ____________

Reteaching (continued)

Conditional Statements

To write the contrapositive of a conditional, negate the original hypothesis and conclusion *and also* switch the hypothesis and conclusion.

Conditional: If the weather is rainy, then the sidewalks are wet.

Contrapositive: If the sidewalks are not wet, then the weather is not rainy.

The conditional and its contrapositive are always both true or both false, so they have the same truth value. The converse and the inverse also have the same truth value. However, the conditional and contrapositive may have a different truth value than the converse and the inverse.

Exercises

Identify the hypothesis and conclusion of each conditional.

1. If a number is a multiple of 2, then the number is even.
   Hypothesis: A number is a multiple of 2. Conclusion: The number is even.
2. If something is thrown up into the air, then it must come back down.
   Hypothesis: Something is thrown up into the air. Conclusion: It must come back down.
3. Two angles are supplementary if the angles form a linear pair.
   Hypothesis: Two angles form a linear pair. Conclusion: The angles are supplementary.
4. If the shoe fits, then you should wear it.
   Hypothesis: The shoe fits. Conclusion: You should wear it.

State whether each conditional is *true* or *false.* Write the converse for the conditional and state whether the converse is *true* or *false.*

5. If the recipe uses 3 teaspoons of sugar, then it uses 1 tablespoon of sugar.
   True; if the recipe uses 1 tablespoon of sugar, then it uses 3 teaspoons of sugar; true.
6. If the milk has passed its expiration date, then the milk should not be consumed. **True; if the milk should not be consumed, then the milk has passed its expiration date; false.**

State whether each conditional is *true* or *false.* Write the inverse for the conditional and state whether the inverse is *true* or *false.*

7. If the animal is a fish, then it lives in water.
   True; if the animal is not a fish, then the animal does not live in water; false.
8. If your car tires are not properly inflated, then you will get lower gas mileage.
   True; if your car tires are inflated properly, then you will not get lower gas mileage; true.

State whether each conditional is *true* or *false.* Write the contrapositive for the conditional and state whether the contrapositive is *true* or *false.*

9. If you ride on a roller coaster, then you will experience sudden drops.
   True; if you do not experience sudden drops, then you are not riding a roller coaster; true.
10. If you only have $15, then you can buy a meal that costs $15.65.
    False; if you cannot buy a meal that costs $15.65, then you do not only have $15; false.

Name ______________________ Class ____________ Date ____________

Reteaching

Biconditionals and Definitions

If a conditional statement and its converse are both true, we say the original conditional is reversible. It works both ways because both $p \rightarrow q$ and $q \rightarrow p$ are true.

If a conditional is reversible, you can write it as a **biconditional**. A biconditional uses the words "if and only if." A biconditional can be written as $p \leftrightarrow q$.

Conditional: If a triangle has three congruent sides, then the triangle is equilateral.

Converse: If a triangle is equilateral, then the triangle has three congruent sides.

The conditional is true. The converse is also true. Since the conditional and its converse are both true, the original statement is "reversible" and the biconditional will be true.

Biconditional: A triangle has three congruent sides if and only if it is an equilateral triangle.

Exercises

Test each statement below to see if it is reversible. If it is reversible, write it as a true biconditional. If not, write *not reversible.*

1. If a whole number is a multiple of 2, then the whole number is even.
Reversible; a whole number is a multiple of 2 if and only if the whole number is even.

2. Rabbits are animals that eat carrots.
not reversible

3. Two lines that intersect to form four 90° angles are perpendicular.
Reversible; two lines intersect to form four 90° angles if and only if the two lines are perpendicular.

4. Mammals are warm-blooded animals.
not reversible

Write the two conditionals that form each biconditional.

5. A parallelogram is a rectangle if and only if the diagonals are congruent.
If a parallelogram is a rectangle, then the diagonals are congruent. If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

6. An animal is a giraffe if and only if the animal's scientific name is *Giraffa camelopardalis.*
If an animal is a giraffe, then the animal's scientific name is *Giraffa camelopardalis*. If the animal's scientific name is *Giraffa camelopardalis*, then the animal is a giraffe.

Name ______________________________________ Class _____________ Date _____________

Reteaching (continued)

Biconditionals and Definitions

A good definition:

- is reversible; the statement works both ways
- can be written as a true biconditional
- avoids using vague, imprecise, or difficult words

Problem

Is the following a good definition for a square?

Definition: A square is a rectangle with four congruent sides.

Is the definition reversible? Yes.

A rectangle with four congruent sides is a square.

Can the definition be rewritten as a biconditional? Yes.

A figure is a square if and only if it is a rectangle with four congruent sides.

Is the definition clear and understandable? Yes.

The definition is a good definition for a square.

Exercises

State whether each statement is a good definition. Explain your answer.

7. A parallelogram is a quadrilateral with two pairs of parallel sides.
The definition is good. The statement is reversible. It can be written as a true biconditional and is clear and understandable.

8. A triangle is a three-sided figure whose angle measures sum to 180°.
The definition is good. The statement is reversible. It can be written as a true biconditional and is clear and understandable.

9. A juice drink is a beverage that contains less than 100% juice.
The definition is not good. The statement is not reversible because a beverage that contains 0% juice, such as water, should not be called a juice drink.

10. In basketball, the top scorer in a game is the player who scores the most points in the game.
The definition is good. The statement is reversible and can be written as a true biconditional and is clear and understandable.

11. A tree is a large, green, leafy plant.
The definition is not good because the term “large” is vague. There are large, green, leafy plants that are not trees.

Name ______________________________ Class ____________ Date ____________

Reteaching

Deductive Reasoning

The **Law of Detachment** states that if a conditional statement is true, then any time the conditions for the hypothesis exist, the conclusion is true.

If the conditional statement is not true, or the conditions of the hypothesis do not exist, then you cannot make a valid conclusion.

Problem

What can you conclude from the following series of statements?

If an animal has feathers and can fly, then it is a bird.
A crow has feathers and can fly.

Is the conditional statement true? Yes.

Do the conditions of the hypothesis exist? Yes.

Therefore, you can conclude that a crow is a bird.

If an animal has feathers and can fly, then it is a bird.
A bat can fly.

Is the conditional statement true? Yes.

Do the conditions of the hypothesis exist? No; a bat does not have feathers.

Therefore, no conclusions can be made with the given information.

Exercises

Use the Law of Detachment to make a valid conclusion based on each conditional. Assume the conditional statement is true.

1. If it is Monday, then Jim has tae kwon do practice.
The date is Monday, August 25. **Conclusion: Jim has tae kwon do practice.**

2. If the animal is a whale, then the animal lives in the ocean.
Daphne sees a beluga whale. **Conclusion: The beluga whale lives in the ocean.**

3. If you live in the city of Miami, then you live in the state of Florida.
Jani lives in Florida. **Conclusion: No conclusion is possible.**

4. If a triangle has an angle with a measure greater than 90, then the triangle is obtuse.
In $\triangle GHI$, $m\angle HGI = 110$. **Conclusion: $\triangle GHI$ is obtuse.**

5. A parallelogram is a rectangle if its diagonals are congruent.
Lincoln draws a parallelogram on a piece of paper. **Conclusion: No conclusion is possible.**

Name ______________________________ Class ____________ Date ____________

Reteaching (continued)

Deductive Reasoning

You can use the **Law of Syllogism** to string together two or more conditionals and draw a conclusion based on the conditionals.

Notice that the conclusion of each conditional becomes the hypothesis of the next conditional.

$$\left.\begin{array}{c} p \rightarrow q \\ q \rightarrow r \\ r \rightarrow s \end{array}\right\} \text{Therefore: } p \rightarrow s$$

Problem

What can you conclude from the following two conditionals?

If a polygon is a hexagon, then the sum of its angle measures is 720.
If a polygon's angle measures sum to 720, then the polygon has six sides.

The conclusion of the first conditional matches the hypothesis of the second conditional: the sum of the angle measures of a polygon is 720.

You can conclude that if a polygon is a hexagon, then it has six sides.

Exercises

If possible, use the Law of Syllogism to make a conclusion. If it is not possible to make a conclusion, tell why.

6. If you are climbing Pikes Peak, then you are in Colorado. If you are in Colorado, then you are in the United States. **Conclusion: If you are climbing Pikes Peak, then you are in the United States.**

7. If the leaves are falling from the trees, then it is fall.

If it is September 30, then it is fall. **No valid conclusion is possible. The two statements stand alone.**

8. If it is spring, then the leaves are coming back on the trees.

If it is April 14, then it is spring. **Conclusion: If it is April 14, then the leaves are coming back on the trees.**

9. If plogs plunder, then flegs fret.

If flegs fret, then gops groan. **Conclusion: If plogs plunder, then gops groan.**

10. If you make a 95 on the next test, you will pass the course.

If you pass the course, you will not have to take summer school. **Conclusion: If you make a 95 on the next test, then you will not have to take summer school.**

11. Use the Law of Detachment and the Law of Syllogism to make a valid conclusion from the following series of true statements. Explain your reasoning and which statements you used in your reasoning.

I. If an animal is an insect, then it has a head, thorax, and abdomen.

II. If an animal is a spider, then it has a head and abdomen.

III. If an animal has a head, thorax, and abdomen, then it has six legs.

IV. Humberto saw an insect called a grasshopper.

Sample: Conclusion: The grasshopper has a head, a thorax, an abdomen, and six legs. I used the Law of Detachment, the Law of Syllogism, and statements I, III, and IV.

Name ______________________________ Class ____________ Date ____________

Reteaching

Reasoning in Algebra and Geometry

When you solve equations you use the Properties of Equality.

Property	Words	Example
Addition Property	You can add the same number to each side of an equation.	If $x = 2$, then $x + 2 = 4$.
Subtraction Property	You can subtract the same number from each side of an equation.	If $y = 8$, then $y - 3 = 5$.
Multiplication Property	You can multiply by the same number on each side of an equation.	If $z = 2$, then $5z = 10$.
Division Property	You can divide each side of an equation by the same number.	If $6m = 12$, then $m = 2$.
Substitution Property	You can exchange a part of an expression with an equivalent value.	If $3x + 5 = 3$ and $x = 2$, then $3(2) + 5 = 3$.

Exercises

Support each conclusion with a reason.

1. Given: $6x + 2 = 12$

   Conclusion: $6x = 10$
   Subtraction Property

2. Given: $m\angle 1 + m\angle 2 = 90$

   Conclusion: $m\angle 1 = 90 - m\angle 2$
   Subtraction Property

3. Given: $x = m\angle C$

   Conclusion: $2x = m\angle C + x$
   Addition Property

4. Given: $q - x = r$

   Conclusion: $4(q - x) = 4r$
   Multiplication Property

5. Given: $m\angle Q - m\angle R = 90$,

   $m\angle Q = 4m\angle R$

   Conclusion: $4m\angle R - m\angle R = 90$
   Substitution Property

6. Given: $CD = AF - 2CD$

   Conclusion: $3CD = AF$
   Addition Property

7. Given: $5(y - x) = 20$

   Conclusion: $5y - 5x = 20$
   Distributive Prop.

8. Given: $m\angle AOX = 2m\angle XOB$

   $2m\angle XOB = 140$

   Conclusion: $m\angle AOX = 140$
   Substitution Prop.

9. Order the steps below to complete the proof.

   Given: $m\angle P + m\angle Q = 90$, $m\angle Q = 5m\angle P$

   Prove: $m\angle Q = 75$ d, a, c, b

   a) $6m\angle P = 90$ by the Distributive Property
   b) $m\angle Q = 5 \cdot 15 = 75$ by the Substitution and Multiplication Properties
   c) $m\angle P = 15$ by the Division Property
   d) $m\angle P + 5m\angle P = 90$ by the Substitution Property

Name ______________________ Class ____________ Date ____________

Reteaching (continued)

Reasoning in Algebra and Geometry

Several other important properties are also needed to write proofs.

Example	Words
Reflexive Property of Equality $AB = AB$	Any value is equal to itself.
Reflexive Property of Congruence $\angle Z \cong \angle Z$	Any geometric object is congruent to itself.
Symmetric Property of Equality If $XY = ZA$, then $ZA = XY$.	You can change the order of an equality.
Symmetric Property of Congruence If $\angle Q \cong \angle R$, then $\angle R \cong \angle Q$.	You can change the order of a congruence statement.
Transitive Property of Equality If $KL = MN$ and $MN = TR$, then $KL = TR$.	If two values are equal to a third value, then they are equal to each other.
Transitive Property of Congruence If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.	If two values are congruent to a third value, then they are congruent to each other.

Exercises

Match the property to the appropriate statement.

10. $\overline{RT} \cong \overline{RT}$ b

11. If $\angle YER \cong \angle IOP$ f
and $\angle IOP \cong \angle WXZ$
then $\angle YER \cong \angle WXZ$

12. If $\overline{PQ} \cong \overline{MN}$ d
then $\overline{MN} \cong \overline{PQ}$

13. If $XT = YZ$ e
and $YZ = UP$
then $XT = UP$

14. $m\angle 1 = m\angle 1$ a

15. If $m\angle RQS = m\angle TEF$ c
then $m\angle TEF = m\angle RQS$

a) Reflexive Property of Equality

b) Reflexive Property of Congruence

c) Symmetric Property of Equality

d) Symmetric Property of Congruence

e) Transitive Property of Equality

f) Transitive Property of Congruence

16. Writing Write six new mathematical statements that represent each of the properties given above.
Check students' work.

Name ______________________ Class ____________ Date ____________

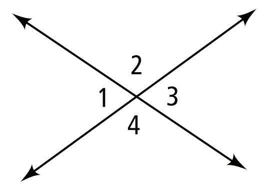
Reteaching

Proving Angles Congruent

A *theorem* is a conjecture or statement that you prove true using deductive reasoning. You prove each step using any of the following: given information, definitions, properties, postulates, and previously proven theorems.

The proof is a chain of logic. Each step is justified, and then the Laws of Detachment and Syllogism connect the steps to prove the theorem.

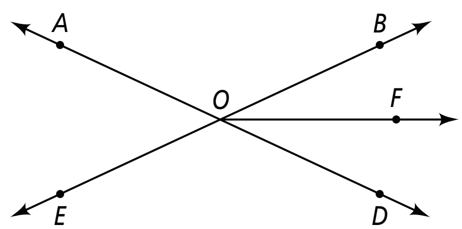
Vertical angles are angles on opposite sides of two intersecting lines. In the figure at the right, $\angle 1$ and $\angle 3$ are vertical angles. $\angle 2$ and $\angle 4$ are also vertical angles. The Vertical Angles Theorem states that vertical angles are always congruent. The symbol $\cong$ means *is congruent to*.



Problem

Given: $m\angle BOF = m\angle FOD$

Prove: $2m\angle BOF = m\angle AOE$



Statements	Reasons
1) $m\angle BOF = m\angle FOD$	1) Given
2) $m\angle BOF + m\angle FOD = m\angle BOD$	2) Angle Addition Postulate
3) $m\angle BOF + m\angle BOF = m\angle BOD$	3) Substitution Property
4) $2(m\angle BOF) = m\angle BOD$	4) Combine like terms.
5) $\angle AOE \cong \angle BOD$	5) Vertical Angles are $\cong$.
6) $m\angle AOE = m\angle BOD$	6) Definition of Congruence
7) $2m\angle BOF = m\angle AOE$	7) Substitution Property

Exercises

Write a paragraph proof.

1. Given: $\angle AOB$ and $\angle XOZ$ are vertical angles.

$m\angle AOB = 80$

$m\angle XOZ = 6x + 5$

Prove: $x = 12.5$

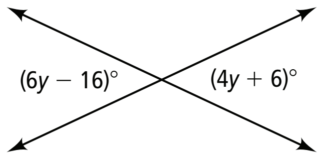
Because it is given that $\angle AOB$ and $\angle XOZ$ are vertical angles, and all vertical angles are $\cong$, it follows that $m\angle AOB = m\angle XOZ$. Because it is also given that $m\angle AOB = 80$ and $m\angle XOZ = 6x + 5$, the use of substitution yields the equation $80 = 6x + 5$. Using the Subtraction Property of Equality, $75 = 6x$, and using the Division Property of Equality, it follows that $x = 12.5$.

Reteaching (continued)

Proving Angles Congruent

Find the value of each variable.

2.



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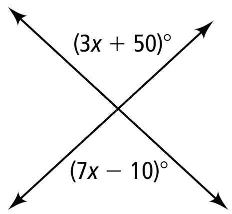
3.



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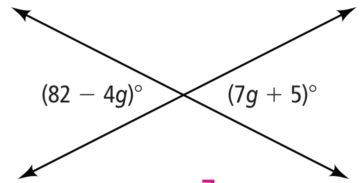

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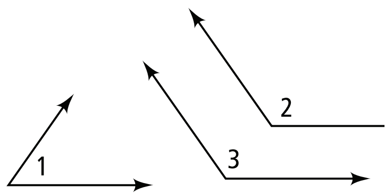
You can use numbers to help understand theorems that may seem confusing.

Congruent Supplements Theorem: If two angles are supplements of the same angle (or of congruent angles), then the two angles are congruent.



Think about it: Suppose $m\angle 1 = 50$. Any angle supplementary to $\angle 1$ must have a measure of 130. So, supplements of $\angle 1$ must be congruent.

If $\angle 2$ and $\angle 3$ are both supplementary to $\angle 1$, then $\angle 2 \cong \angle 3$.

Congruent Complements Theorem: If two angles are complements of the same angle (or of congruent angles), then the two angles are congruent.

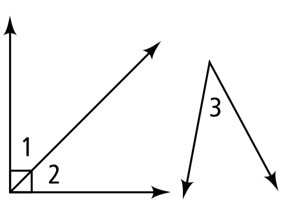
If $\angle 4$ and $\angle 5$ are both complementary to $\angle 6$, then $\angle 4 \cong \angle 5$.

Think about it: Suppose $m\angle 6 = 30$. Any complement of $\angle 6$ has a measure of 60. So, all complements of $\angle 6$ must be congruent.

Exercises

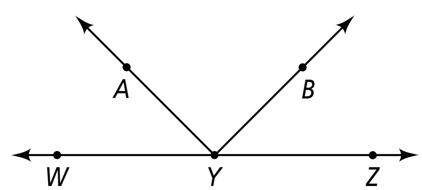
Name a pair of congruent angles in each figure. Justify your answer.

5. Given: $\angle 2$ is complementary to $\angle 3$.



$\angle 1 \cong \angle 3$; Congruent Complements Theorem

6. Given: $\angle AYZ \cong \angle BYW$



$\angle AYW \cong \angle BYZ$; Congruent Supplements Theorem

7. Reasoning Explain why the following statement is true. Use numbers in your explanation. "If $\angle 1$ is supplementary to $\angle 2$, $\angle 2$ is supplementary to $\angle 3$, $\angle 3$ is supplementary to $\angle 4$, and $\angle 4$ is supplementary to $\angle 5$, then $\angle 1 \cong \angle 5$."

Suppose $m\angle 1 = 50$; therefore $m\angle 2 = 130$, $m\angle 3 = 50$, $m\angle 4 = 130$, and $m\angle 5 = 50$. Because $m\angle 1 = m\angle 5$, $\angle 1 \cong \angle 5$.