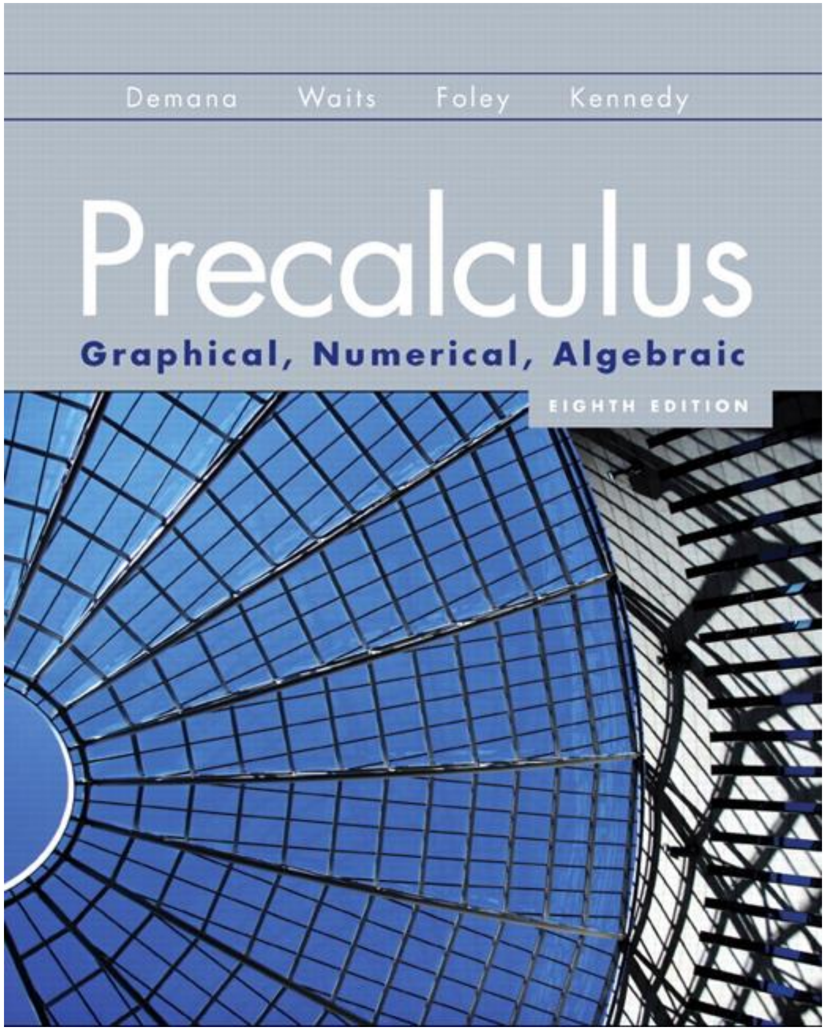
4.8

Solving Problems with Trigonometry



What you'll learn about

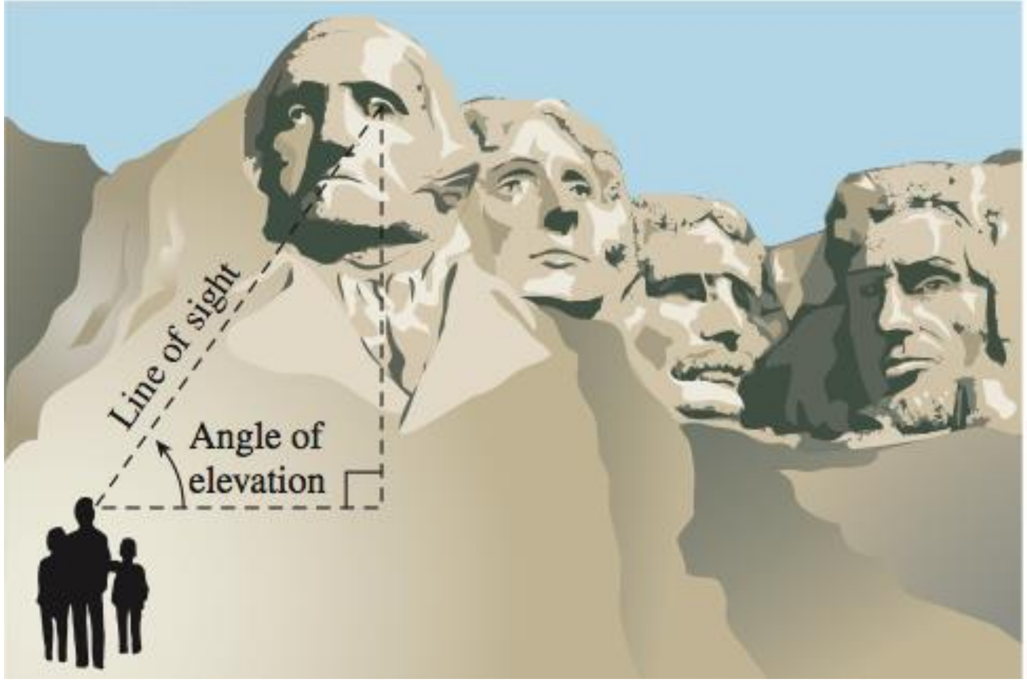
- More Right Triangle Problems
- Simple Harmonic Motion

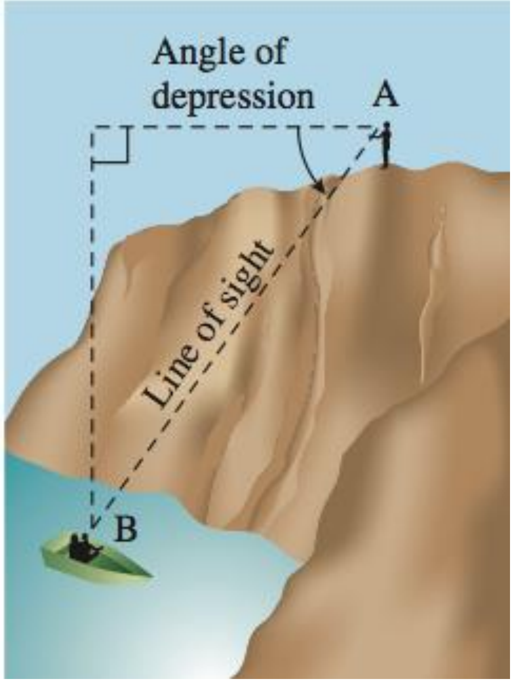
… and why

These problems illustrate some of the better-known applications of trigonometry.

Angle of Elevation, Angle of Depression

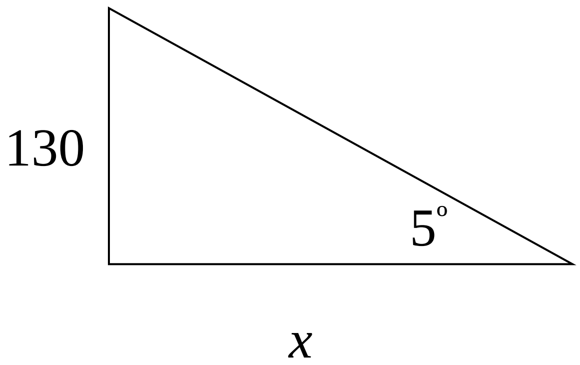
An **angle of elevation** is the angle through which the eye moves up from horizontal to look at something above. An **angle of depression** is the angle through which the eye moves down from horizontal to look at something below.





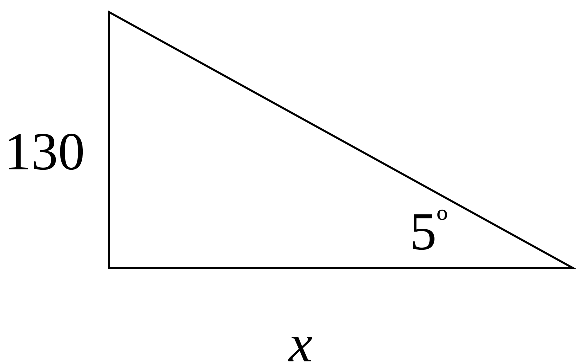
Example Using Angle of Elevation

The angle of elevation from the buoy to the top of the Barnegat Bay lighthouse 130 feet above the surface of the water is 5°. Find the distance x from the base of the lighthouse to the buoy.



Example Using Angle of Elevation

The angle of elevation from the buoy to the top of the Barnegat Bay lighthouse 130 feet above the surface of the water is 5°. Find the distance *x* from the base of the lighthouse to the buoy.



$$\tan 5^\circ = \frac{130}{x}$$

$$x = \frac{130}{\tan 5^\circ} \approx 1485.9$$

The buoy is about 1486 feet from the base of the lighthouse.

Example **Making Indirect Measurements**

From the top of a 100 ft. tall air traffic control tower, an airplane is observed flying toward the tower. If the angle of elevation of the airplane changes from 32° to 10° during the period of observation, and the altitude of the airplane changes from 500 ft to 200 ft, how far does the plane travel?

Example **Making Indirect Measurements**

The figure models the situation.

Let x = distance from the tower at the second observation.

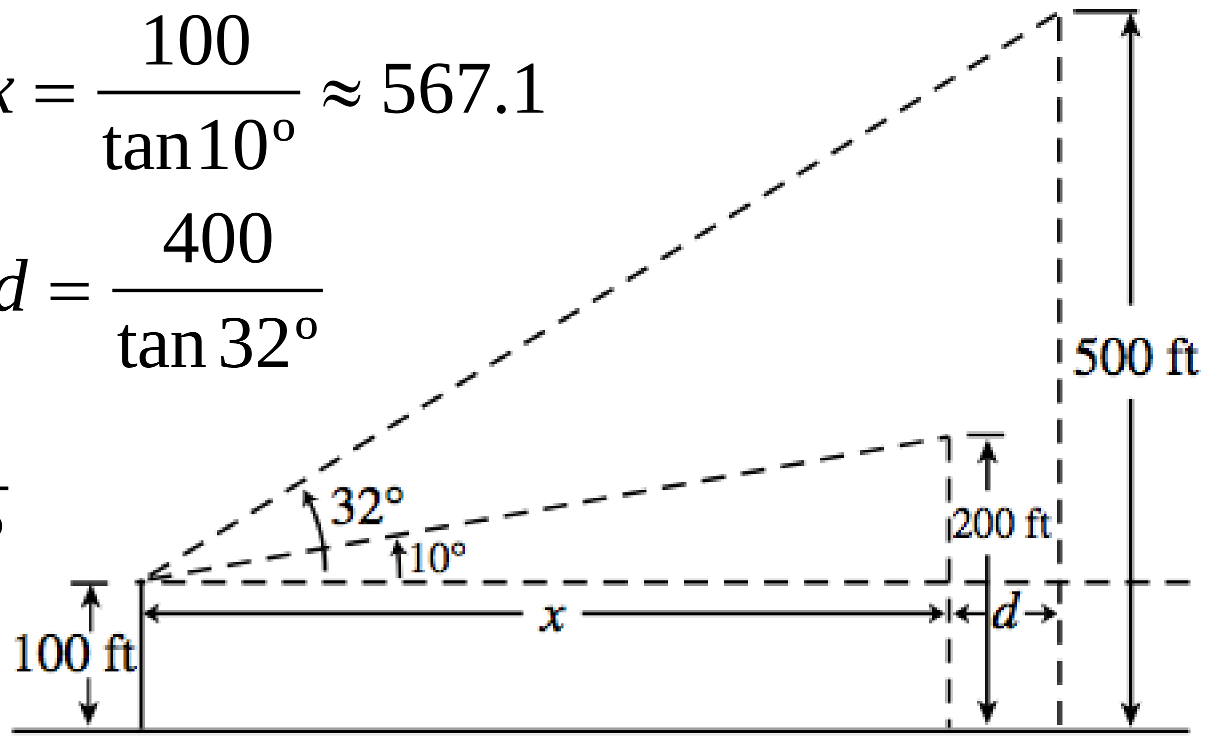
Let d = distance plan moves

$$\tan 10^\circ = \frac{100}{x} \qquad x = \frac{100}{\tan 10^\circ} \approx 567.1$$

$$\tan 32^\circ = \frac{400}{x+d} \quad x + d = \frac{400}{\tan 32^\circ}$$

$$d = \frac{400}{\tan 32^\circ} - \frac{100}{\tan 10^\circ}$$

$$d \approx 73.0$$



Simple Harmonic Motion

A point moving on a number line is in **simple harmonic motion** if its directed distance d from the origin is given by either $d = a\sin\omega t$ or $d = a\cos\omega t$, where a and ω are real numbers and $\omega > 0$.
The motion has frequency $\omega / 2\pi$, which is the number of oscillations per unit of time.

Example Calculating Harmonic Motion

A mass oscillating up and down on the bottom of a spring (assuming perfect elasticity and no friction or air resistance) can be modeled as harmonic motion. If the weight is displaced a maximum of 4 cm, find the modeling equation if it takes 3 seconds to complete one cycle.

Example Calculating Harmonic Motion

Assume the spring is at the origin of the coordinate system when $t = 0$ and use the equation $d = a\sin\omega t$.

The maximum displacement is 4 cm, so $a = 4$.

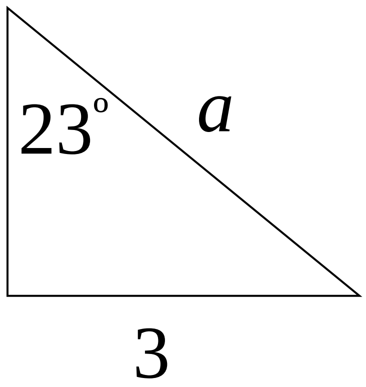
One cycle takes 3 sec, so the period is 3 and the frequency is 1/3.

Therefore, $\dfrac{\omega}{2\pi} = \dfrac{1}{3}$ and $\omega = \dfrac{2\pi}{3}$.

Put this together and $d = a\sin\omega t \Rightarrow d = 4\sin\left(\dfrac{2\pi}{3}t\right)$.

Quick Review

1. Solve for *a*.

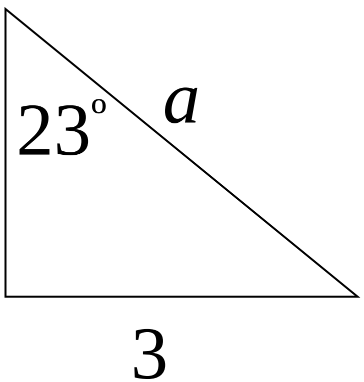


Quick Review

2. Find the complement of 47°.
3. Find the supplement of 47°.
4. State the bearing that describes the direction NW (northwest).
5. State the amplitude and period of the sinusoid $3\cos 2(x+1)$.

Quick Review Solutions

1. Solve for *a*.



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Quick Review Solutions

2. Find the complement of 47°. 43°

3. Find the supplement of 47°. 133°

4. State the bearing that describes the direction NW (northwest). 135°

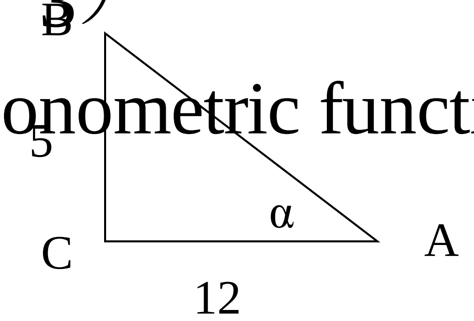
5. State the amplitude and period of the sinusoid $3\cos 2(x+1)$. $A = 3,\ p = \pi$

Chapter Test

1. The point $(-1, \sqrt{3})$ is on the terminal side of an angle in standard position. Give the smallest positive angle measure in both degrees and radians.

2. Evaluate $\sec\left(-\frac{\pi}{3}\right)$ without using a calculator.

3. Find all six trigonometric functions of α in VABC.



Chapter Test

4. The point $(-5, -3)$ is on the terminal side of angle θ. Evaluate the six trigonometric functions for θ.
5. Use transformations to describe how the graph of the function $y = -2 - 3\sin(x - \pi)$ is related to the function $y = \sin x$. Graph two periods.
6. State the amplitude, period, phase shift, domain and range for $f(x) = 1.5\sin(2x - \pi/4)$.

Chapter Test

7. Find the exact value of x without using a calculator: $\tan x = -1, \quad 0 \le x \le \pi.$

8. Describe the end behavior of $f(x) = \dfrac{\sin x}{x^2}.$

9. Find an algebraic expression equivalent to $\tan\left(\cos^{-1} x\right).$

10. From the top of a 150-ft building Kana observes a car moving toward her. If the angle of depression of the car changes from 18° to 42° during the observation, how far does the car travel?

Chapter Test Solutions

1. The point $(-1, \sqrt{3})$ is on the terminal side of an angle in standard position. Give the smallest positive angle measure in both degrees and radians.

$120^{\circ} = 2\pi/3$ radians

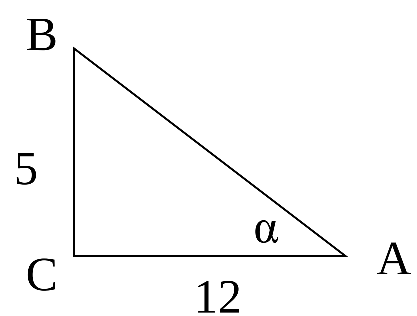
2. Evaluate $\sec\left(-\dfrac{\pi}{3}\right)$ without using a calculator. 2

3. Find all six trigonometric functions of α in VABC.

$\sin\alpha = 5/13$ $\csc\alpha = 13/5$

$\cos\alpha = 12/13$ $\sec\alpha = 13/12$

$\tan\alpha = 5/12$ $\cot\alpha = 12/5$



Chapter Test Solutions

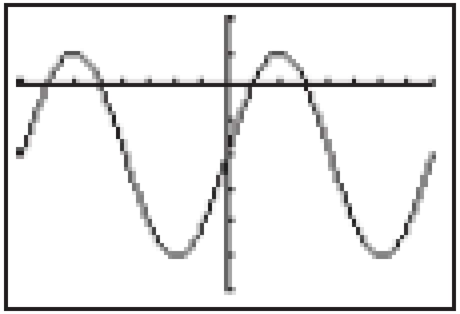
4. The point (–5,–3) is on the terminal side of angle θ.
   Evaluate the six trigonometric functions for θ.

$\sin\theta = -3/\sqrt{34}$; $\csc\theta = -\sqrt{34}/3$; $\cos\theta = -5/\sqrt{34}$;

$\sec\theta = -\sqrt{34}/5$; $\tan\theta = 3/5$; $\cot\theta = 5/3$

5. Use transformations to describe how the graph of the function $y = -2 - 3\sin(x - \pi)$ is related to the function $y = \sin x$. Graph two periods.

translation right π units, vertical stretch by a factor of 3, reflected across the x-axis, translation down 2 units.



$[-2\pi, 2\pi]$ by $[-6, 2]$

Chapter Test Solutions

6. State the amplitude, period, phase shift, domain and range for $f(x) = 1.5\sin\left(2x - \pi/4\right)$.

   A=1.5; p=π; ps=π/8; domain:$(-\infty,\infty)$; range:$[-1.5,1.5]$

7. Find the exact value of x without using a calculator: $\tan\, x = -1,\ \ 0 \le x \le \pi$. $3\pi/4$

8. Describe the end behavior of $f(x) = \dfrac{\sin x}{x^2}$.

   As $|x| \to \infty,\ f(x) \to 0$

Chapter Test Solutions

9\. Find an algebraic expression equivalent to $\tan\left(\cos^{-1} x\right)$.

$$\frac{\sqrt{1-x^2}}{x}$$

10\. From the top of a 150-ft building Kana observes a car moving toward her. If the angle of depression of the car changes from 18° to 42° during the observation, how far does the car travel? $150(\cot 18^{\circ} - \cot 42^{\circ}) \approx 295$ ft