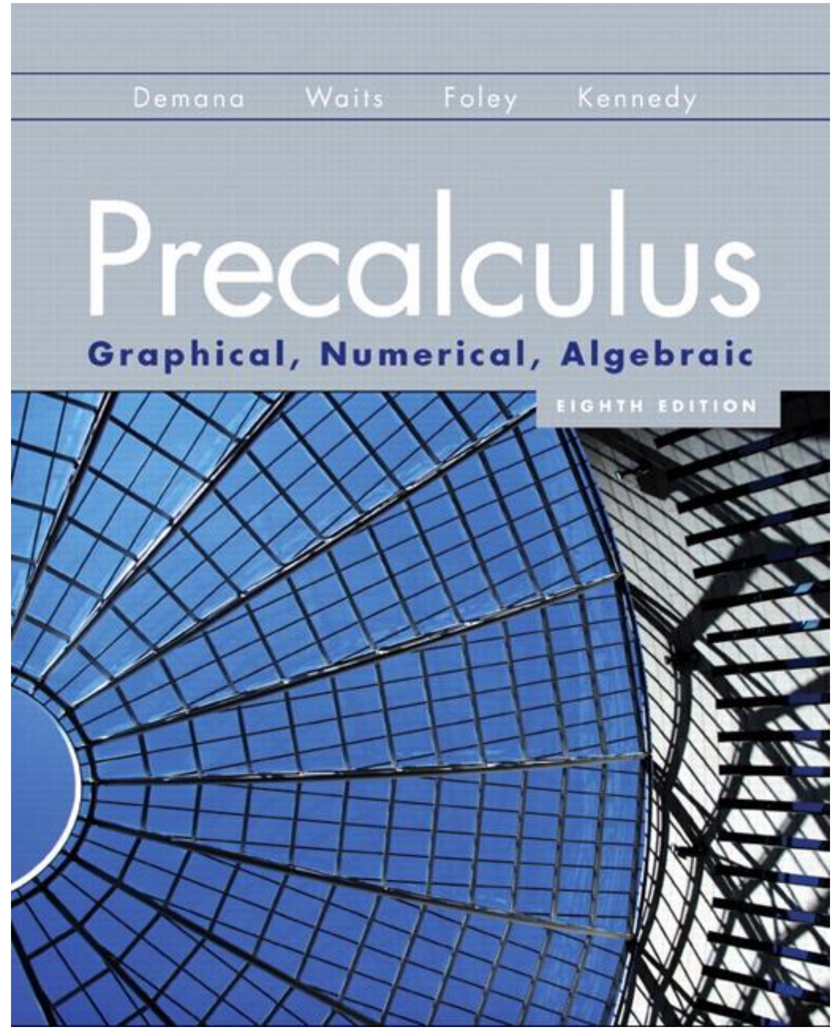
4.7

Inverse Trigonometric Functions



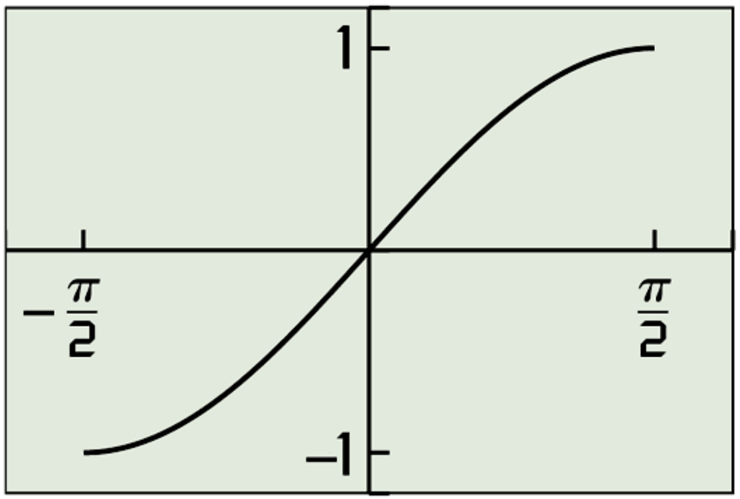
What you'll learn about

- Inverse Sine Function
- Inverse Cosine and Tangent Functions
- Composing Trigonometric and Inverse Trigonometric Functions
- Applications of Inverse Trigonometric Functions

… and why

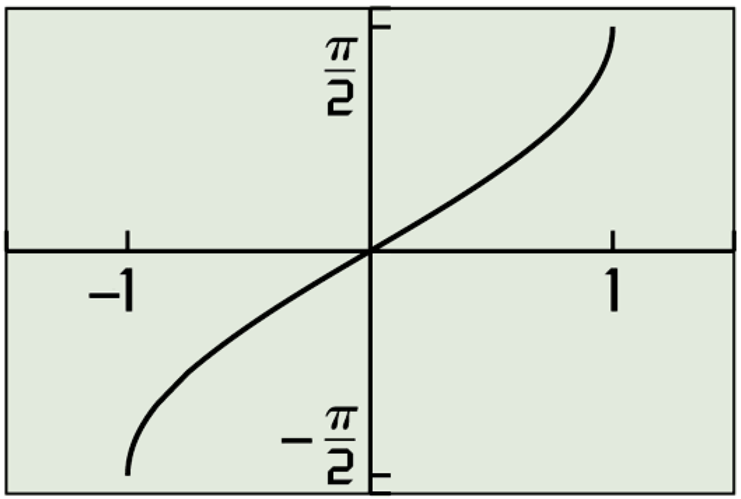
Inverse trig functions can be used to solve trigonometric equations.

Inverse Sine Function



[–2, 2] by [–1.2, 1.2]

(a)



[–1.5, 1.5] by [–1.7, 1.7]

(b)

Inverse Sine Function (Arcsine Function)

The unique angle y in the interval $\left[-\pi/2, \pi/2\right]$ such that $\sin y = x$ is the **inverse sine** (or **arcsine**) of x, denoted $\mathbf{sin^{-1}} \boldsymbol{x}$ or **arcsin** $\boldsymbol{x}$.

The domain of $y = \sin^{-1} x$ is $\left[-1, 1\right]$ and the range is $\left[-\pi/2, \pi/2\right]$.

Example **Evaluate $\sin^{-1} x$ Without a Calculator**

Find the exact value without a calculator: $\sin^{-1}\left(-\frac{1}{2}\right)$

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Find the point on the right half of the unit circle whose y-coordinate is $-1/2$ and draw a reference triangle. Recognize this as a special ratio, and the angle in the interval $[-\pi/2, \pi/2]$ whose sin is $-1/2$ is $-\pi/6$.

Example **Evaluate sin^{-1}*x* Without a Calculator**

Find the exact value without a calculator: $\sin^{-1}\left(\sin\left(\frac{\pi}{10}\right)\right)$.

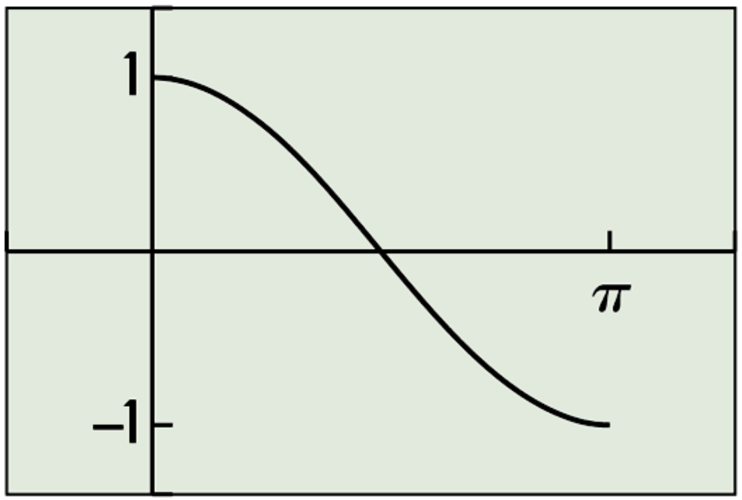
Example **Evaluate $\sin^{-1}x$ Without a Calculator**

Find the exact value without a calculator: $\sin^{-1}\left(\sin\left(\frac{\pi}{10}\right)\right)$.

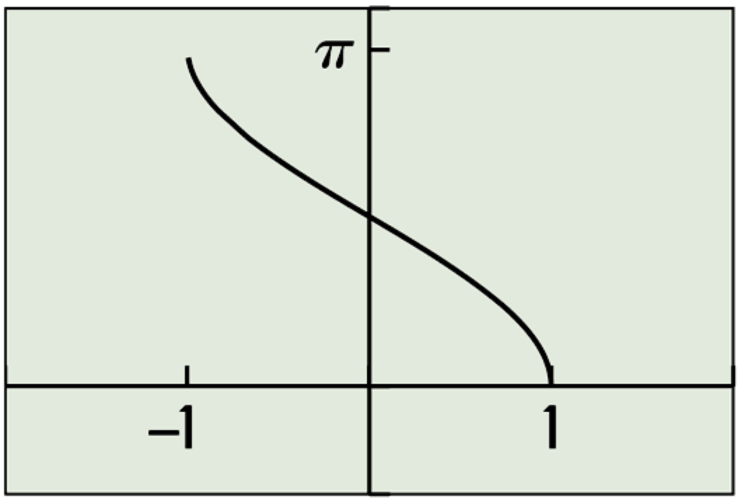
Draw an angle $\pi/10$ in standard position and mark its y-coordinate on the y-axis. The angle in the interval $[-\pi/2, \pi/2]$ whose sine is this number is $\pi/10$.

Therefore, $\sin^{-1}\left(\sin\left(\frac{\pi}{10}\right)\right) = \frac{\pi}{10}$.

Inverse Cosine (Arccosine Function)



[–1, 4] by [–1.4, 1.4]
(a)



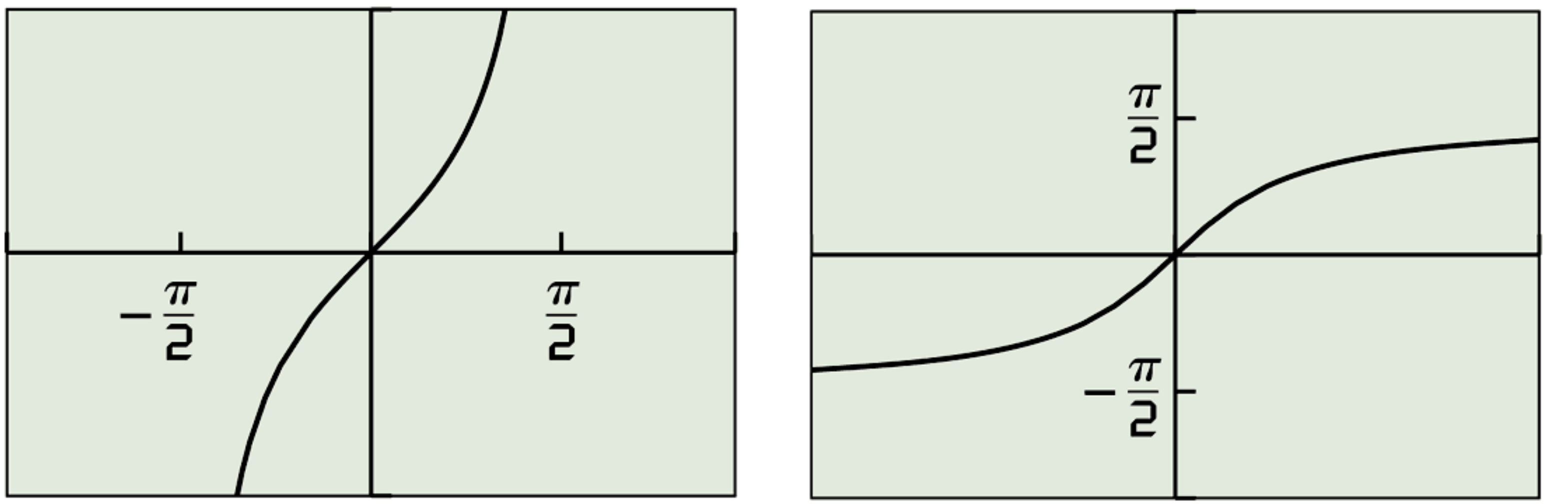
[–2, 2] by [–1, 3.5]
(b)

Inverse Cosine (Arccosine Function)

The unique angle y in the interval $[0, \pi]$ such that $\cos y = x$ is the **inverse cosine** (or **arccosine**) of x, denoted $\mathbf{cos}^{-1}\, \boldsymbol{x}$ or **arccos** $\boldsymbol{x}$.

The domain of $y = \cos^{-1} x$ is $[-1, 1]$ and the range is $[0, \pi]$.

Inverse Tangent Function (Arctangent Function)



[–3, 3] by [–2, 2]
(a)

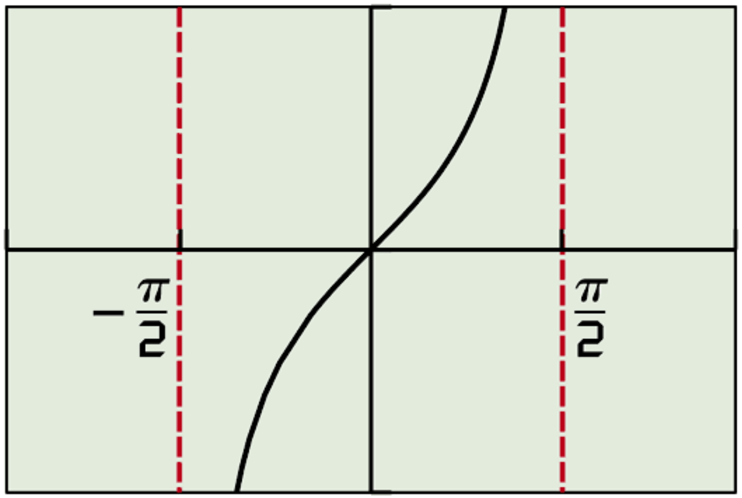
[–4, 4] by [–2.8, 2.8]
(b)

Inverse Tangent Function (Arctangent Function)

The unique angle y in the interval $(-\pi/2, \pi/2)$ such that $\tan y = x$ is the **inverse tangent** (or **arctangent**) of x, denoted $\mathbf{\tan^{-1}} \boldsymbol{x}$ or **arctan *x***.

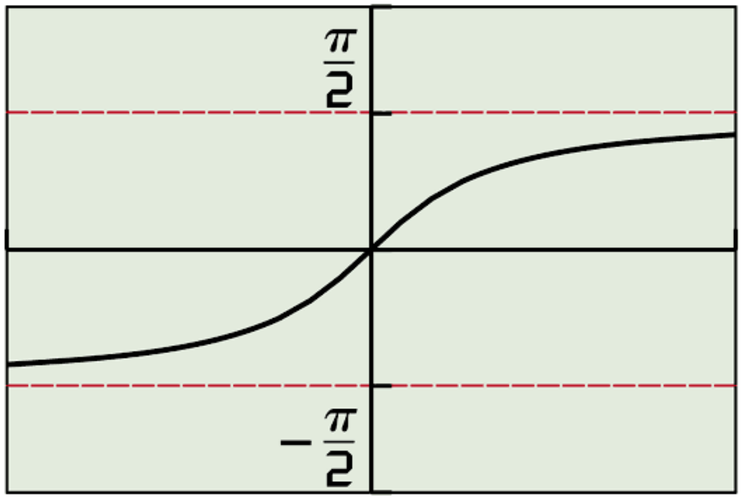
The domain of $y = \tan^{-1} x$ is $(-\infty, \infty)$ and the range is $(-\pi/2, \pi/2)$.

End Behavior of the Tangent Function



[–3, 3] by [–2, 2]

(a)



[–4, 4] by [–2.8, 2.8]

(b)

Composing Trigonometric and Inverse Trigonometric Functions

The following equations are always true whenever they are defined:

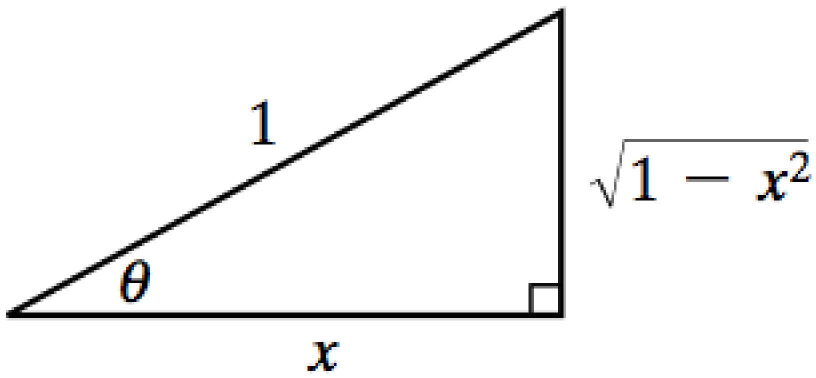
$$\sin\left(\sin^{-1}(x)\right) = x \quad \cos\left(\cos^{-1}(x)\right) = x \quad \tan\left(\tan^{-1}(x)\right) = x$$

The following equations are only true for x values in the "restricted" domains of sin, cos, and tan:

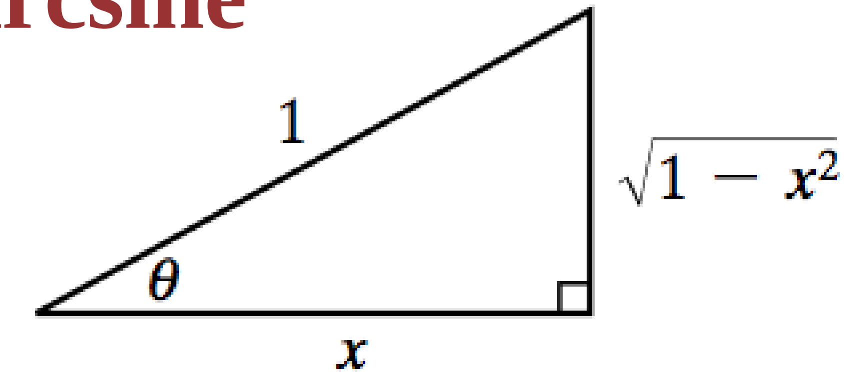
$$\sin^{-1}\left(\sin(x)\right) = x \quad \cos^{-1}\left(\cos(x)\right) = x \quad \tan^{-1}\left(\tan(x)\right) = x$$

Example **Composing Trig Functions with Arcsine**

Compose each of the six basic trig functions with $\cos^{-1} x$ and reduce the composite function to an algebraic expression involving no trig functions.



Example Composing Trig Functions with Arcsine



Use the triangle to find the required ratios:

$$\sin(\cos^{-1} x)) = \sqrt{1 - x^2}$$

$$\csc(\cos^{-1} x)) = \frac{1}{\sqrt{1 - x^2}}$$

$$\cos(\cos^{-1} x)) = x$$

$$\sec(\cos^{-1} x)) = \frac{1}{x}$$

$$\tan(\cos^{-1} x)) = \frac{\sqrt{1 - x^2}}{x}$$

$$\cot(\cos^{-1} x)) = \frac{x}{\sqrt{1 - x^2}}$$

Quick Review

State the sign (positive or negative) of the sine, cosine, and tangent in quadrant

1. I
2. III

Find the exact value.

3. $\cos \dfrac{\pi}{6}$

4. $\tan \dfrac{4\pi}{3}$

5. $\sin \left(-\dfrac{11\pi}{6}\right)$

Quick Review Solutions

State the sign (positive or negative) of the sine, cosine, and tangent in quadrant

1. I $+,+,+$
2. III $-,-,+$

Find the exact value.

3. $\cos \dfrac{\pi}{6}$ $\sqrt{3}/2$
4. $\tan \dfrac{4\pi}{3}$ $\sqrt{3}$
5. $\sin -\dfrac{11\pi}{6}$ $1/2$