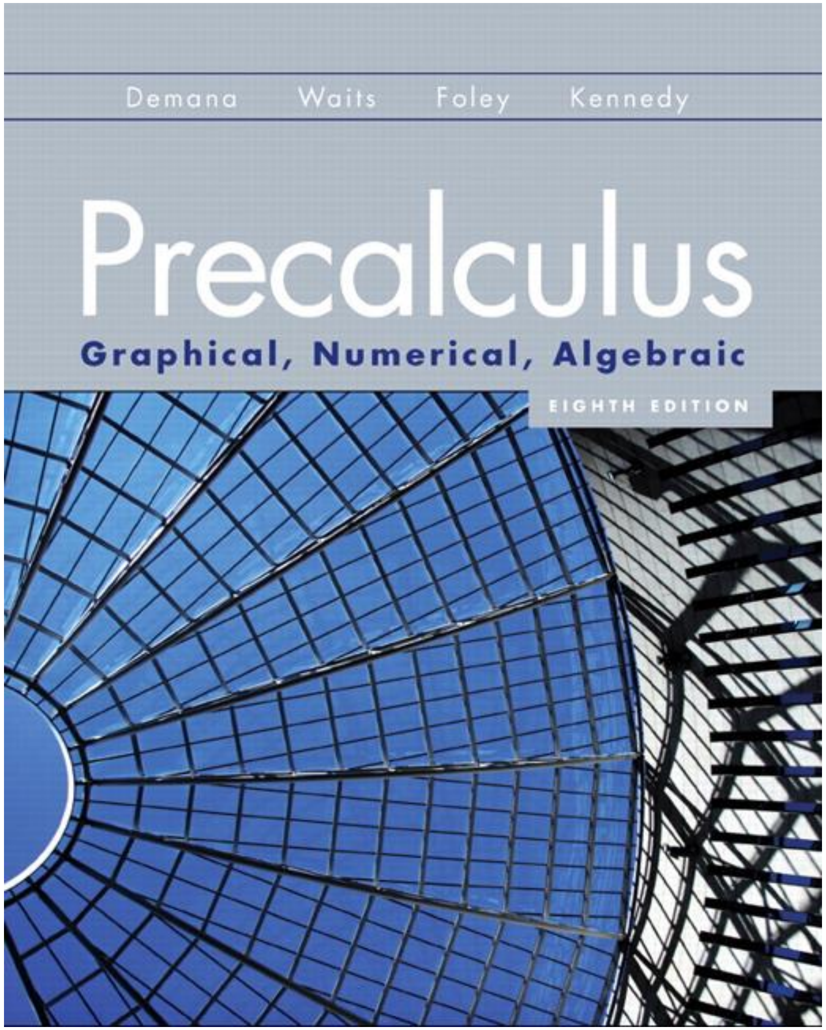
# 4.6

# Graphs of Composite Trigonometric Functions



# What you'll learn about

- Combining Trigonometric and Algebraic Functions
- Sums and Differences of Sinusoids
- Damped Oscillation

… and why

Function composition extends our ability to model periodic phenomena like heartbeats and sound waves.

# Example **Combining the Cosine Function with $x^2$**

Graph $y = (\cos x)^2$ and state whether the function appears to be periodic.

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QuickTime™ and a
decompressor
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Graph $y = \cos\left(x^2\right)$ and state whether the function appears to be periodic.

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Graph $y = \cos\left(x^2\right)$ and state whether the function appears to be periodic.

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The function appears not to be periodic.

# Example **Adding a Sinusoid to a Linear Function**

Graph $f(x) = \cos x - \dfrac{x}{3}$ and state its domain and range.

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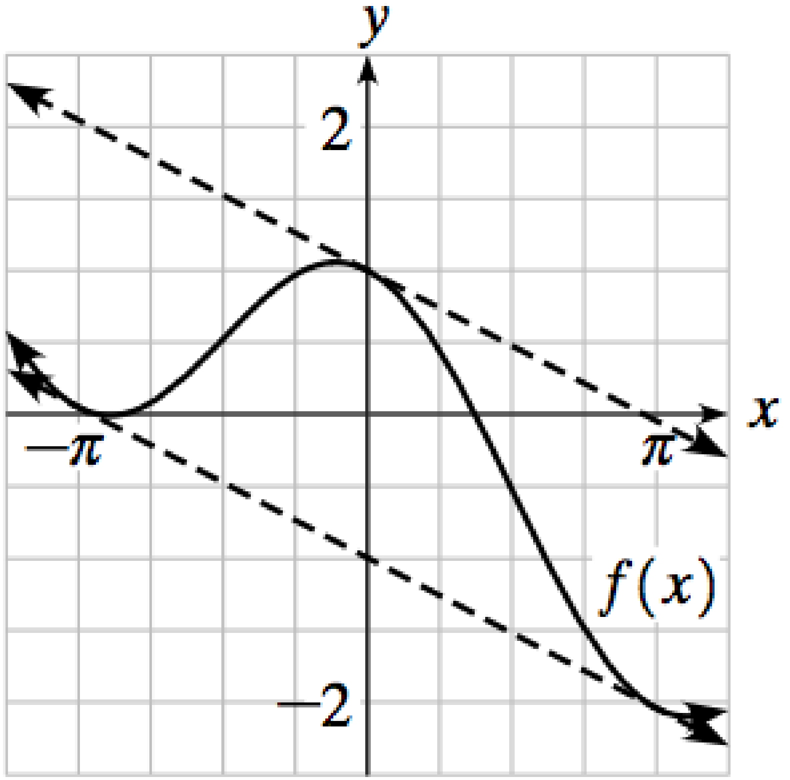
Graph $f(x) = \cos x - \frac{x}{3}$ and state its domain and range.

The function $f$ is the sum of the functions $g(x) = \cos x$ and $h(x) = -\frac{x}{3}$.

Here's the graph of $f = g + h$.

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$



# Sums That Are Sinusoids Functions

If $y_1 = a_1 \sin(b(x - h_1))$ and $y_2 = a_2 \cos(b(x - h_2))$, then

$$y_1 + y_2 = a_1 \sin(b(x - h_1)) + a_2 \cos(b(x - h_2))$$

is a sinusoid with period $2\pi/|b|$.

# Example **Identifying a Sinusoid**

Determine whether the following function is or is not a sinusoid.

$$f(x) = 3\cos x + 5\sin x$$

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Yes, since both functions in the sum have period $2\pi$.

# Example **Identifying a Sinusoid**

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$$f(x) = \cos 3x \ + \ \sin 5x$$

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No, since $\cos 3x$ has period $2\pi / 3$ and $\sin 5x$ has period $2\pi / 5$.

# Damped Oscillation

The graph of $y = f(x)\cos bx$ (or $y = f(x)\sin bx$) oscillates between the graphs of $y = f(x)$ and $y = -f(x)$. When this reduces the amplitude of the wave, it is called **damped oscillation**. The factor $f(x)$ is called the **damping factor**.

# Quick Review

State the domain and range of the function.

1. $f(x) = -3\sin 2x$
2. $f(x) = |x| + 2$
3. $f(x) = 2\cos 3x$
4. Describe the behavior of $y = e^{-3x}$ as $x \to \infty$.
5. Find $f \circ g$ and $g \circ f$, given $f(x) = x^2 + 3$ and $g(x) = \sqrt{x}$

# Quick Review Solutions

State the domain and range of the function.

1. $f(x) = -3\sin 2x$ Domain: $(-\infty, \infty)$ Range: $[-3, 3]$

2. $f(x) = |x| + 2$ Domain: $(-\infty, \infty)$ Range: $[2, \infty)$

3. $f(x) = 2\cos 3x$ Domain: $(-\infty, \infty)$ Range: $[-2, 2]$

4. Describe the behavior of $y = e^{-3x}$ as $x \to \infty$. $\lim_{x \to \infty} e^{-3x} = 0$

5. Find $f \circ g$ and $g \circ f$, given $f(x) = x^2 + 3$ and $g(x) = \sqrt{x}$

$f \circ g = x + 3;\ g \circ f = \sqrt{x^2 + 3}$