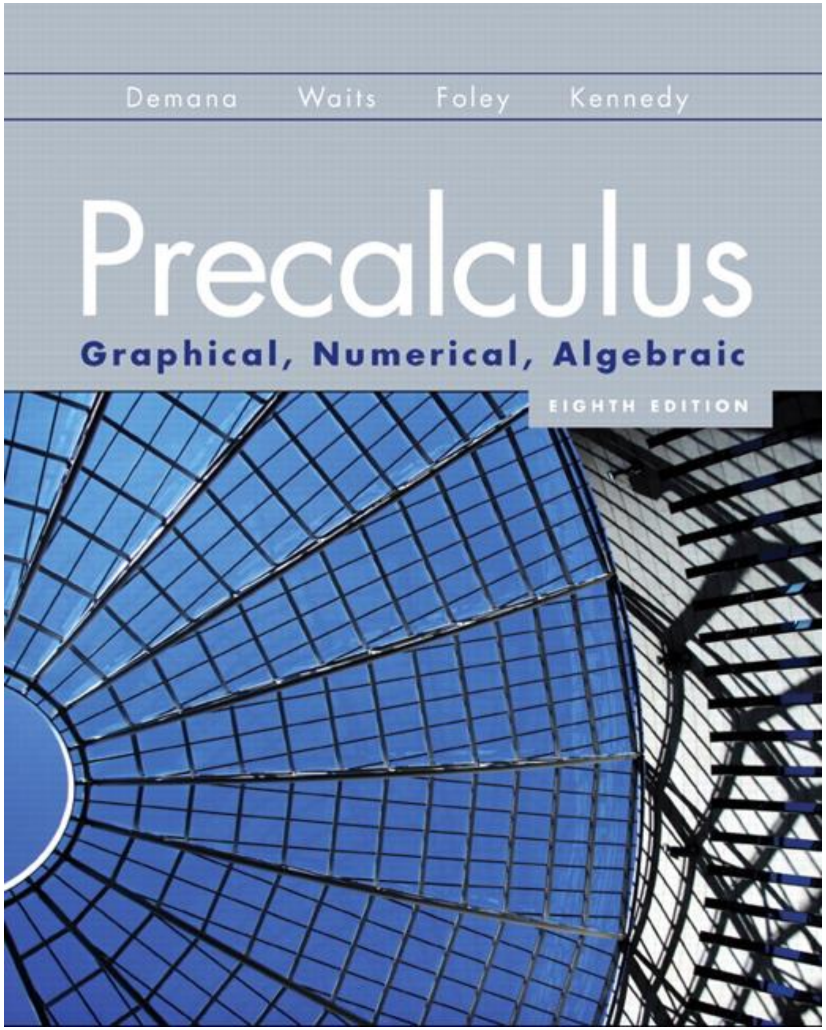
4.4

Graphs of Sine and Cosine: Sinusoids



What you'll learn about

- The Basic Waves Revisited
- Sinusoids and Transformations
- Modeling Periodic Behavior with Sinusoids

… and why

Sine and cosine gain added significance when used to model waves and periodic behavior.

Sinusoid

A function is a **sinusoid** if it can be written in the form $f(x) = a\sin(bx + c) + d$ where a, b, c, and d are constants and neither a nor b is 0.

Amplitude of a Sinusoid

The **amplitude** of the sinusoid $f(x) = a\sin(bx + c) + d$ is $|a|$.
Similarly, the amplitude of $f(x) = a\cos(bx + c) + d$ is $|a|$.
Graphically, the amplitude is half the height of the wave.

Period of a Sinusoid

The **period** of the sinusoid $f(x) = a\sin(bx + c) + d$ is $2\pi/|b|$.

Similarly, the period of $f(x) = a\cos(bx + c) + d$ is $2\pi/|b|$.

Graphically, the period is the length of one full cycle of the wave.

Example **Horizontal Stretch or Shrink and Period**

Find the period of $y = \sin\left(\dfrac{x}{2}\right)$ and use the language of transformations to describe how the graph relates to $y = \sin x$.

Example Horizontal Stretch or Shrink and Period

Find the period of $y = \sin\left(\frac{x}{2}\right)$ and use the language of transformations to describe how the graph relates to $y = \sin x$.

The period is $\dfrac{2\pi}{\frac{1}{2}} = 4\pi$. The graph of $y = \sin\left(\frac{x}{2}\right)$ is a horizontal stretch of $y = \sin x$ by a factor of 2.

Frequency of a Sinusoid

The **frequency** of the sinusoid $f(x) = a\sin(bx + c) + d$ is $|b|/2\pi$.

Similarly, the frequency of $f(x) = a\cos(bx + c) + d$ is $|b|/2\pi$.

Graphically, the frequency is the number of complete cycles the wave completes in a unit interval.

Example **Combining a Phase Shift with a Period Change**

Construct a sinusoid with period $\pi/3$ and amplitude 4 that goes through $(2,0)$.

Example Combining a Phase Shift with a Period Change

Construct a sinusoid with period $\pi/3$ and amplitude 4 that goes through $(2,0)$.

To find the coefficient of x, set $2\pi/|b| = \pi/3$ and solve for b.

Find $b = \pm 6$. Arbitrarily choose $b = 6$.

For the amplitude set $|a| = 4$. Arbitrarily choose $a = 4$.

The graph contains (2,0) so shift the function 2 units to the right.

$$y = 4\sin(6(x-2)) = 4\sin(6x-12).$$

Example Combining a Phase Shift with a Period Change

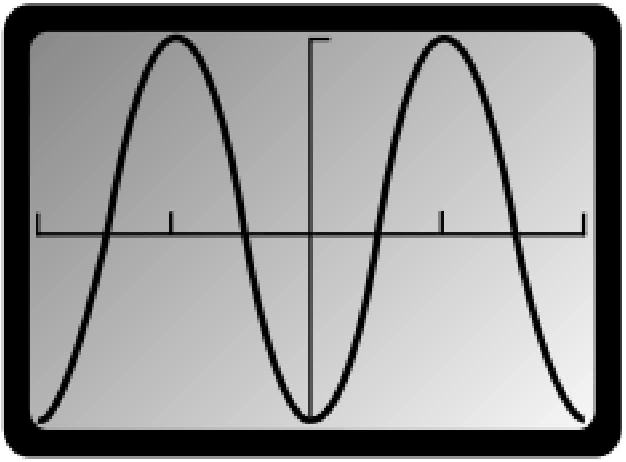
Find the frequency of the function $f(x) = -\frac{1}{3}\cos 5x$ and interpret its meaning graphically. Sketch the graph in the window $\left[-\frac{2\pi}{5}, \frac{2\pi}{5}\right]$ by $\left[-\frac{1}{3}, \frac{1}{3}\right]$.

Example Combining a Phase Shift with a Period Change

The frequency is $5 \div 2\pi = \dfrac{5}{2\pi}$.

This is the reciprocal of the period, which is $2\pi/5$.

The graph completes one cycle per interval of length $2\pi/5$.



$\left[-\dfrac{2\pi}{5}, \dfrac{2\pi}{5}\right]$ by $\left[-\dfrac{1}{3}, \dfrac{1}{3}\right]$

Graphs of Sinusoids

The graphs of $y = a\sin(b(x-h)) + k$ and $y = a\cos(b(x-h)) + k$ (where $a \neq 0$ and $b \neq 0$) have the following characteristics:

$$\text{amplitude} = |a|;$$

$$\text{period} = \frac{2\pi}{|b|};$$

$$\text{frequency} = \frac{|b|}{2\pi}.$$

Graphs of Sinusoids

When complared to the graphs of $y = a\sin bx$ and $y = a\cos bx$, respectively, they also have the following characteristics:

a phase shift of h; a vertical translation of k.

Constructing a Sinusoidal Model using Time

1. Determine the maximum value M and minimum value m. The amplitude A of the sunusoid will be $A = \frac{M - m}{2}$, and the vertical shift will be $C = \frac{M + m}{2}$.

2. Determine the period p, the time interval of a single cycle of the periodic function. The horizontal shrink (or stretch) will be $B = \frac{2\pi}{p}$.

Constructing a Sinusoidal Model using Time

3. Choose an appropriate sinusoid based on behavior at some given time T. For example, at time T:

$f(t) = A\cos(B(t - T)) + C$ attains a maximum value;

$f(t) = -A\cos(B(t - T)) + C$ attains a minimum value;

$f(t) = A\sin(B(t - T)) + C$ is halfway between a minimum and a maximum value;

$f(t) = -A\sin(B(t - T)) + C$ is halfway between a maximum and a minimum value.

Quick Review

State the sign (positive or negative) of the function in each quadrant.

1. $\sin x$
2. $\cot x$

Give the radian measure of the angle.

3. 150°
4. -135°
5. Find a transformation that will transform the graph of $y_1 = \sqrt{x}$ to the graph of $y_2 = 2\sqrt{x}$.

Quick Review Solutions

State the sign (positive or negative) of the function in each quadrant.

1. $\sin x$ $+,+,-,-$
2. $\cot x$ $+,-,+,-$

Give the radian measure of the angle.

3. 150° $5\pi/6$
4. -135° $-3\pi/4$
5. Find a transformation that will transform the graph of $y_1 = \sqrt{x}$ to the graph of $y_2 = 2\sqrt{x}$. vertically stretch by 2