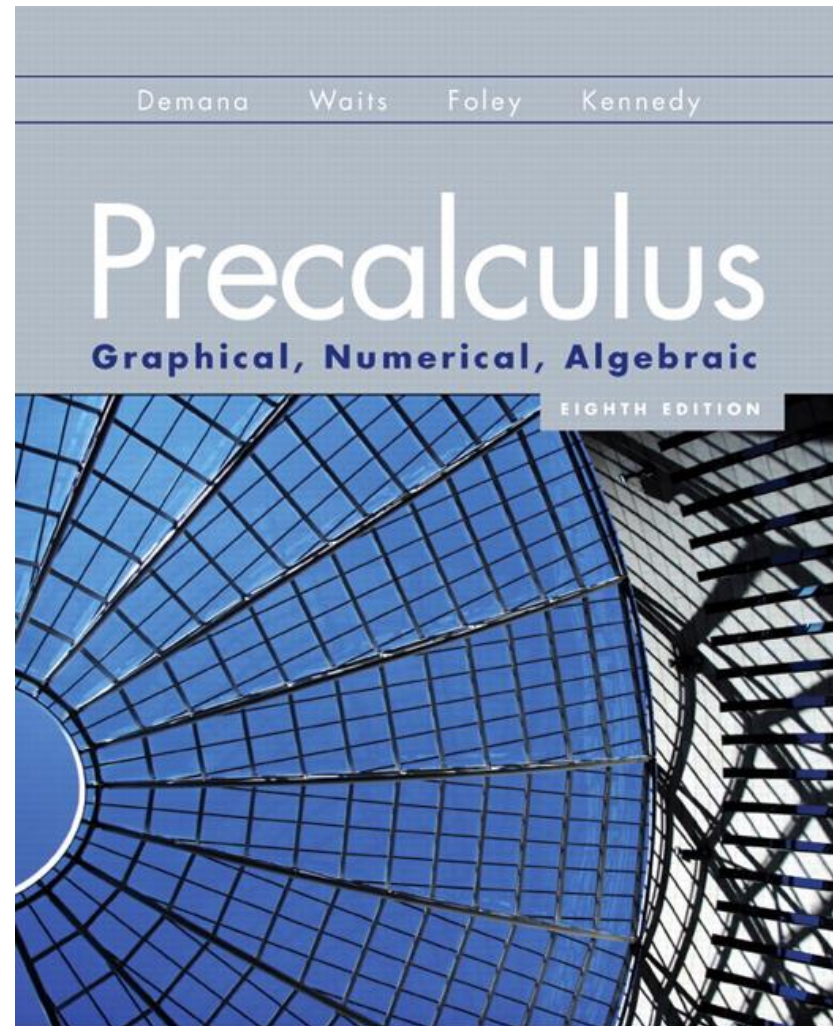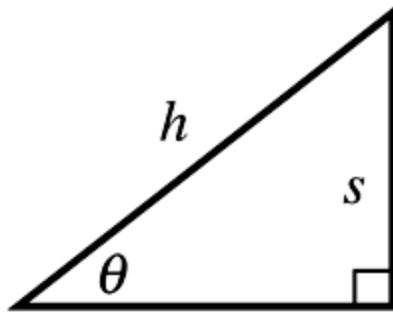# 4.1

Angles and Their Measures

What you'll learn about

- The Problem of Angular Measure
- Degrees and Radians
- Circular Arc Length
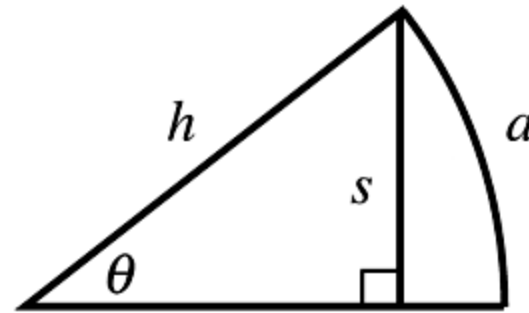- Angular and Linear Motion

… and why

Angles are the domain elements of the trigonometric functions.

Why 360º ?

(a)   (b)

θ is a central angle intercepting a circular arc of length a. The measure can be in degrees (a circle measures 360º once around) or in radians, which measures the length of arc a.
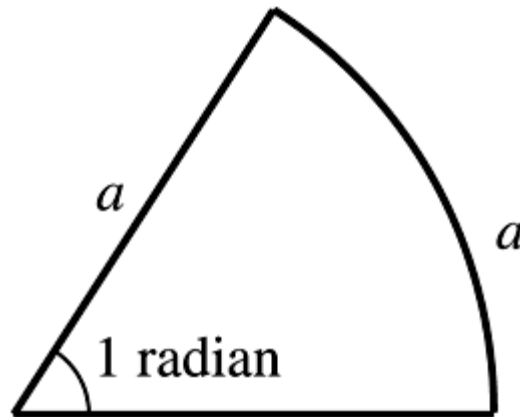
Navigation

In navigation, the **course** or **bearing** of an object is sometimes given as the angle of the **line of travel** measured clockwise from due north.

Radian

A central angle of a circle has measure 1 **radian** if it intercepts an arc with the same length as the radius.

Degree-Radian Conversion

To convert radians to degrees, multiply by

$$\frac{180º}{\pi \text{ radians}}.$$

To convert degrees to radians, multiply by

$$\frac{\pi \text{ radians}}{180º}.$$

Example Working with Degree and Radian Measure

a. How many radians are in 135º?

b. How many degrees are in $\dfrac{7\pi}{6}$ radians?

c. Find the length of an arc intercepted by a central angle of 1/4 radian in a circle of radius 3 in.

Example **Working with Degree and Radian Measure**

a. How many radians are in 135º?

Use the conversion factor $\dfrac{\pi \text{ radians}}{180º}$.

$$135º\,\frac{\pi \text{ radians}}{180º} = \frac{135\pi}{180} \text{ radians} = \frac{3\pi}{4} \text{ radians}$$

Example Working with Degree and Radian Measure

b. How many degrees are in $\dfrac{7\pi}{6}$ radians?

Use the conversion factor $\dfrac{180º}{\pi \text{ radians}}$.

$$\left(\frac{7\pi}{6}\right)\left(\frac{180º}{\pi \text{ radians}}\right) = \frac{1260º}{6} = 210º$$

Example **Working with Degree and Radian Measure**

c. Find the length of an arc intercepted by a central angle of 1/4 radian in a circle of radius 3 in.

A central angle of 1 radian intercepts an arc length of 1 radius, which is 3 in. So a central angle of 1/4 radian intercepts an arc of length 1/4 radius, which is 3/4 in.

Arc Length Formula (Radian Measure)

If θ is a central angle in a circle of radius r, and if θ is measured in radians, then the length s of the intercepted arc is given by

$$s = r\theta.$$

Arc Length Formula (Degree Measure)

If θ is a central angle in a circle of radius r, and if θ is measured in degrees, then the length s of the intercepted arc is given by

$$s = \frac{\pi r \theta}{180}.$$

Example **Perimeter of a Pizza Slice**

Find the perimeter of a 30º slice of a large 8 in. radius pizza.

Example **Perimeter of a Pizza Slice**

Find the perimeter of a 30º slice of a large 8 in. radius pizza.

Let s equal the arc length of the pizza's curved edge.

$$s = \frac{\pi(8)(30)}{180} = \frac{240\pi}{180} \approx 4.2 \text{ in.}$$

$$P = 8 \text{ in.} + 8 \text{ in.} + s \text{ in.}$$
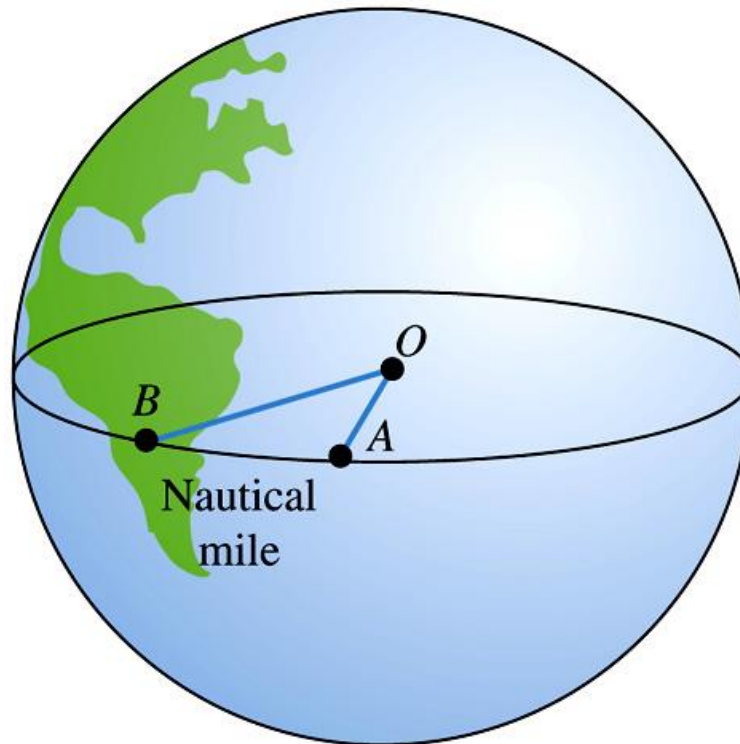
$$P = 20.2 \text{ in.}$$

Angular and Linear Motion

- Angular speed is measured in units like revolutions per minute.

- Linear speed is measured in units like miles per hour.

Nautical Mile

A **nautical mile** (naut mi) is the length of 1 minute of arc along Earth's equator.

Nautical mile

Distance Conversions

1 statute mile ≈ 0.87 nautical mile

1 nautical mile ≈ 1.15 statute mile

Quick Review

1. Find the circumference of the circle with a radius of 4.5 in.

2. Find the radius of the circle with a circumference of 14 cm.

3. Given $s = r\theta$. Find s if $r = 2.2$ cm and $\theta = 4$ radians.

4. Convert 65 miles per hour into feet per second.

5. Convert 9.8 feet per second to miles per hour.

Quick Review Solutions

1. Find the circumference of the circle with a radius of 4.5 in.

   9π in

2. Find the radius of the circle with a circumference of 14 cm.

   $7 / \pi$ cm

3. Given $s = r\theta$. Find s if $r = 2.2$ cm and $\theta = 4$ radians.

   8.8 cm

4. Convert 65 miles per hour into feet per second.

   $95.\overline{3}$ feet per second

5. Convert 9.8 feet per second to miles per hour.

   $6.68\overline{1}$ miles per hour