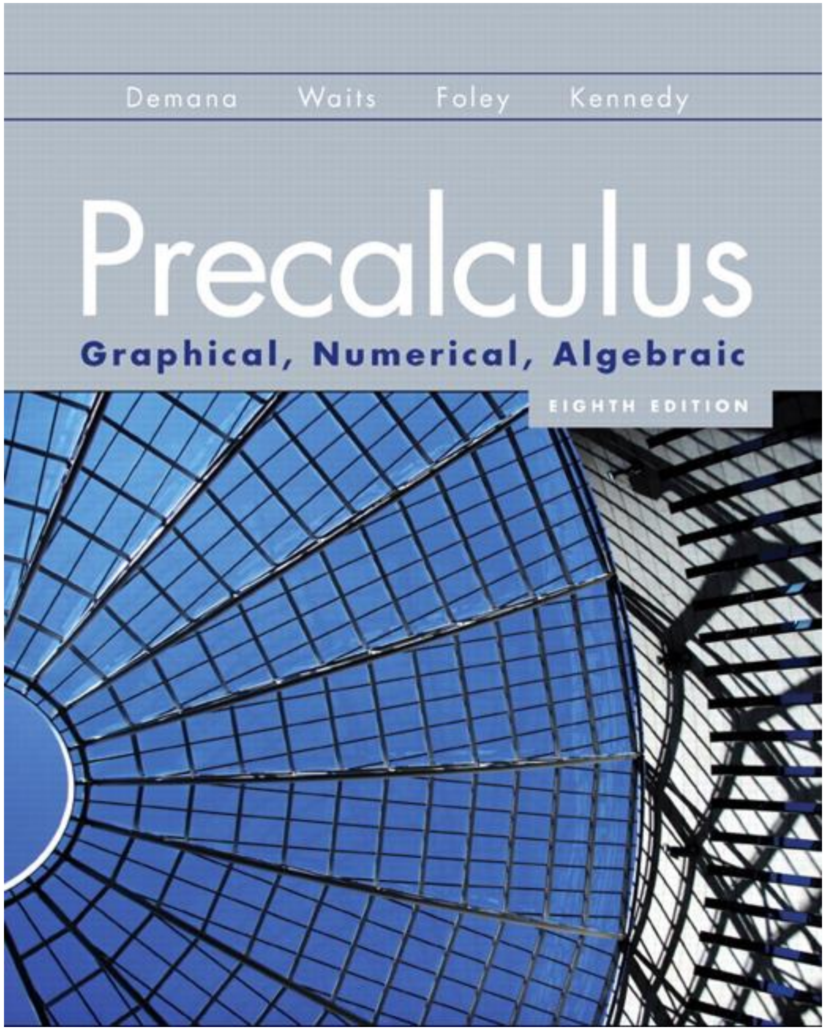
3.6

Mathematics of Finance



PEARSON
Addison Wesley

What you'll learn about

- Interest Compounded Annually
- Interest Compounded *k* Times per Year
- Interest Compounded Continuously
- Annual Percentage Yield
- Annuities – Future Value
- Loans and Mortgages – Present Value

… and why

The mathematics of finance is the science of letting your money work for you – valuable information indeed!

Interest Compounded Annually

If a principal P is invested at a fixed annual interest rate r, calculated at the end of each year, then the value of the investment after n years is

$$A = P(1 + r)^n,$$

where r is expressed as a decimal.

Interest Compounded k Times per Year

Suppose a principal P is invested at an annual rate r compounded k times a year for t years. Then r/k is the interest rate per compounding period, and kt is the number of compounding periods. The amount A in the account after t years is $A = P\left(1+\frac{r}{k}\right)^{kt}$.

Example Compounding Monthly

Suppose Paul invests $400 at 8% annual interest compounded monthly. Find the value of the investment after 5 years.

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Let $P = 400,\ r = 0.08,\ k = 12$, and $t = 5$,

$$A = P\left(1 + \frac{r}{k}\right)^{kt}$$

$$= 400\left(1 + \frac{0.08}{12}\right)^{12(5)}$$

$$= 595.9382...$$

So the value of Paul's investment after 5 years is $595.94.

Compound Interest – Value of an Investment

Suppose a principal P is invested at a fixed annual interest rate r. The value of the investment after t years is

- $A = P\left(1+\dfrac{r}{k}\right)^{kt}$ when interest compounds k times per year,
- $A = Pe^{rt}$ when interest compounds continuously.

Example Compounding Continuously

Suppose Paul invests \$400 at 8% annual interest compounded continuously. Find the value of his investment after 5 years.

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Suppose Paul invests $400 at 8% annual interest compounded continuously. Find the value of his investment after 5 years.

$$P = 400,\ r = 0.08,\ \text{and } t = 5,$$

$$A = Pe^{rt}$$

$$= 400e^{0.08(5)}$$

$$= 596.7298...$$

So Paul's investment is worth $596.73.

Annual Percentage Yield

A common basis for comparing investments is the **annual percentage yield (APY)** – the percentage rate that, compounded annually, would yield the same return as the given interest rate with the given compounding period.

Example Computing Annual Percentage Yield

Suppose you invest $1500 at 6.25% annual interest compounded monthly. What is the equivalent APY?

Example **Computing Annual Percentage Yield**

Suppose you invest \$1500 at 6.25% annual interest compounded monthly. What is the equivalent APY?

Let x = the equivalent APY. The value after one year is $A = 1500(1+x)$.

$$1500(1+x) = 1500\left(1+\frac{0.0625}{12}\right)^{12}$$

$$(1+x) = \left(1+\frac{0.0625}{12}\right)^{12}$$

Example **Computing Annual Percentage Yield**

Suppose you invest $1500 at 6.25% annual interest compounded monthly. What is the equivalent APY?

$$(1+x)=\left(1+\frac{0.0625}{12}\right)^{12}$$

$$x=\left(1+\frac{0.0625}{12}\right)^{12}-1\approx 0.0643$$

The annual percentage yield is 6.43%.

Future Value of an Annuity

The future value FV of an annuity consisting of n equal periodic payments of R dollars at an interest rate i per compounding period (payment interval) is

$$FV = R\frac{(1+i)^n - 1}{i}.$$

Present Value of an Annuity

The present value PV of an annuity consisting of n equal payments of R dollars at an interest rate i per period (payment interval) is

$$PV = R\frac{1-(1+i)^{-n}}{i}.$$

Quick Review

1. Find 3.4% of 70.
2. What is one-third of 6.25%?
3. 30 is what percent of 150?
4. 28 is 35% of what number?
5. How much does Allyson have at the end of 1 year if she invests $400 at 3% simple interest?

Quick Review Solutions

1. Find 3.4% of 70. 2.38

2. What is one-third of 6.25%? $0.0208\overline{3}$

3. 30 is what percent of 150? 20%

4. 28 is 35% of what number? 80

5. How much does Allyson have at the end of 1 year if she invests \$400 at 3% simple interest? \$412

Chapter Test

1. State whether $f(x)=e^{4-x}+2$ is an exponential growth function or an exponential decay function, and describe its end behavior using limits.

   exponential decay; $\lim_{x\to -\infty} f(x)=\infty$, $\lim_{x\to\infty} f(x)=2$

2. Find the exponential function that satisfies the conditions: Initial height = 18 cm, doubling every 3 weeks.

   $f(x)=18\cdot 2^{x/21}$

3. Find the logistic function that satisfies the conditions: Initial value = 12, limit to growth = 30, passing through (2,20). $f(x)\approx 30/(1+1.5e^{-0.55x})$

Chapter Test

4. Describe how to transform the graph of $y = \log_2 x$ into the graph of $h(x) = -\log_2(x-1)+2$. translate right 1 unit, relect across the x-axis, translate up 2 units.

5. Solve for x: $1.05^x = 3$. $x \approx 22.5171$

6. Solve for x: $\ln(3x+4) - \ln(2x+1) = 5$ $x \approx -0.4915$

7. Find the amount A accumulated after investing a principal P for t years at an interest rate r compounded continuously. $A = Pe^{rt}$

Chapter Test

8. The population of Preston is 89,000 and is decreasing by 1.8% each year.

(a) Write a function that models the population as a function of time t.

$$P(t) = 89{,}000(0.982)^t$$

(b) Predict when the population will be 50,000?

31.74 years

Chapter Test

9. The half-life of a certain substance is 1.5 sec. The initial amount of substance is S_0 grams.

(a) Express the amount of substance remaining as a function of time t. $S(t) = S_0\left(\frac{1}{2}\right)^{t/1.5}$

(b) How much of the substance is left after 1.5 sec? $S_0/2$

(c) How much of the substance is left after 3 sec? $S_0/4$

(d) Determine S_0 if there was 1 g left after 1 min.

$1,009,500$ metric tons

Chapter Test

10. If Joenita invests $1500 into a retirement account with an 8% interest rate compounded quarterly, how long will it take this single payment to grow to $3750?

11.57 years