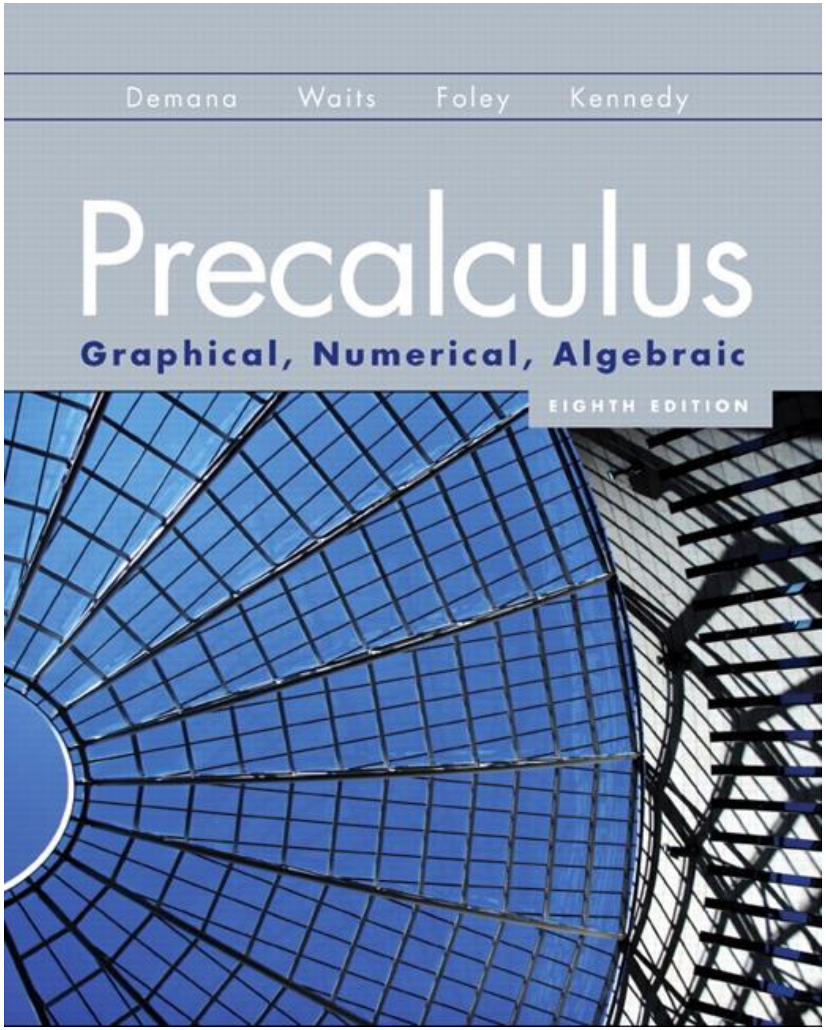
3.4

Properties of Logarithmic Functions



What you'll learn about

- Properties of Logarithms
- Change of Base
- Graphs of Logarithmic Functions with Base *b*
- Re-expressing Data

… and why

The applications of logarithms are based on their many special properties, so learn them well.

Properties of Logarithms

Let $b,\ R$, and S be positve real numbers with $b \neq 1$, and c any real number.

- **Product rule**: $\log_b(RS) = \log_b R + \log_b S$
- **Quotient rule**: $\log_b\left(\frac{R}{S}\right) = \log_b R - \log_b S$
- **Power rule**: $\log_b (R)^c = c \log_b R$

Example **Proving the Product Rule for Logarithms**

Prove $\log_b(RS) = \log_b R + \log_b S.$

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Prove $\log_b (RS) = \log_b R + \log_b S$.

Let $x = \log_b R$ and $y = \log_b S$. The corresponding exponential statements are $b^x = R$ and $b^y = S$. Therefore,

$$RS = b^x \cdot b^y$$

$$RS = b^{x+y}$$

$$\log_b (RS) = x + y \qquad \text{change to logarithmic form}$$

$$\log_b (RS) = \log_b R + \log_b S$$

Example **Expanding the Logarithm of a Product**

Assuming x is positive, use properties of logarithms to write $\log\left(3x^5\right)$ as a sum of logarithms or multiple logarithms.

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Assuming x is positive, use properties of logarithms to write $\log\left(3x^5\right)$ as a sum of logarithms or multiple logarithms.

$$\log\left(3x^5\right) = \log 3 + \log\left(x^5\right)$$
$$= \log 3 + 5\log x$$

Example **Condensing a Logarithmic Expression**

Assuming x is positive, write $3\ln x - \ln 2$ as a single logarithm.

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Assuming x is positive, write $3\ln x - \ln 2$ as a single logarithm.

$$3\ln x - \ln 2 = \ln x^3 - \ln 2$$

$$= \ln\left(\frac{x^3}{2}\right)$$

Change-of-Base Formula for Logarithms

For positive real numbers a, b, and x with $a \neq 1$ and $b \neq 1$,

$$\log_b x = \frac{\log_a x}{\log_a b}.$$

Example Evaluating Logarithms by Changing the Base

Evaluate $\log_3 10$.

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$$\log_3 10 = \frac{\log 10}{\log 3} = \frac{1}{\log 3} \approx 2.096$$

Example Transforming Logarithmic Graphs

Describe how to transform the graph of $f(x) = \ln x$ into the graph of each function.

Graph each function.

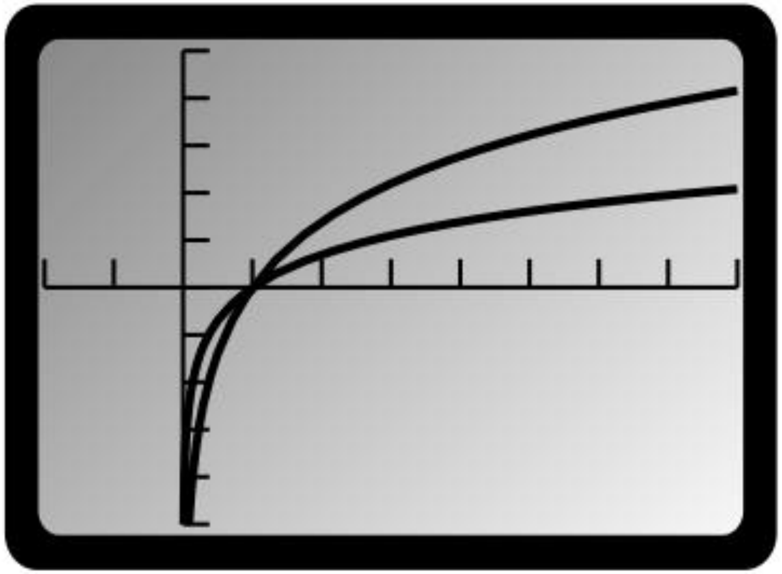
a. $g(x) = \ln x^2$ b. $\text{h}(x) = \log x$ c. $\text{k}(x) = \ln 3x$

Example Transforming Logarithmic Graphs

Describe how to transform the graph of $f(x) = \ln x$ into the graph of each function. Graph each function.

a. $g(x) = \ln x^2$

Since $\ln x^2 = 2 \ln x$, the graph of $g(x) = \ln x^2$ is obtained by vertically stretching the graph of $f(x) = \ln x$ by a factor of 2.



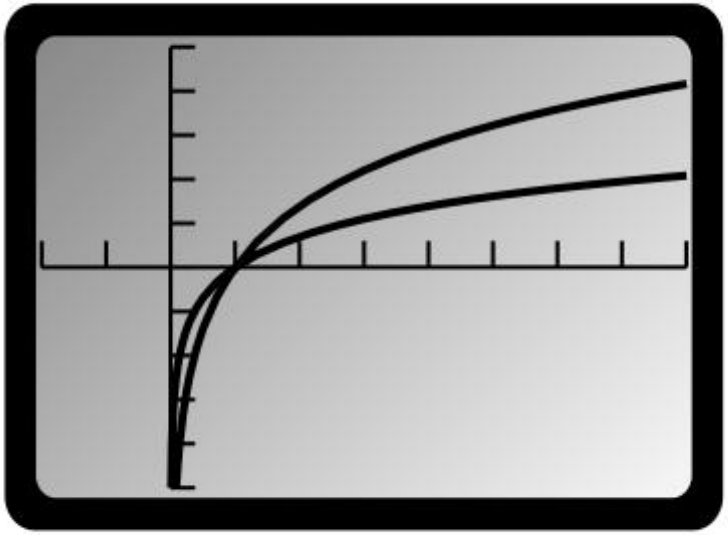
$[-2, 8]$ by $[-5, 5]$

Example Transforming Logarithmic Graphs

Describe how to transform the graph of $f(x) = \ln x$ into the graph of each function. Graph each function.

b. $h(x) = \log x$

Since $\log x = \dfrac{\ln x}{\ln 10}$, the graph of $h(x) = \log x$ is obtained by vertically shrinking the graph of $f(x) = \ln x$ by a factor of $\dfrac{1}{\ln 10} \approx 0.43$



$[-2, 8]$ by $[-5, 5]$

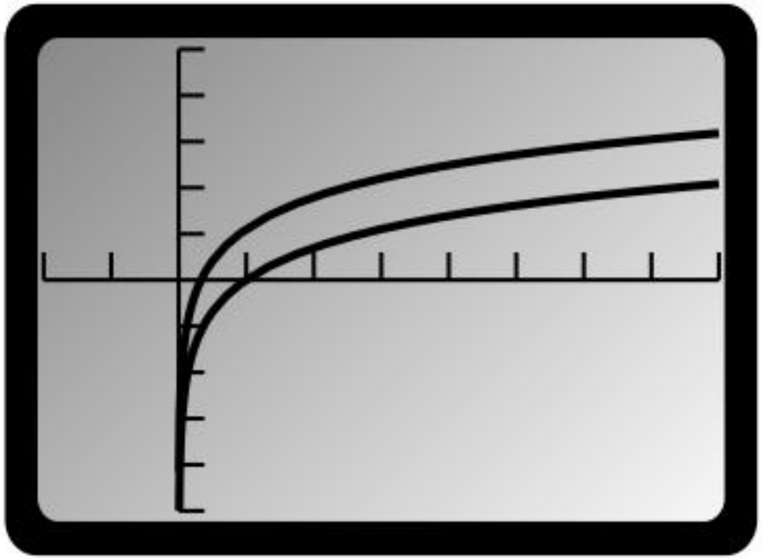
Example Transforming Logarithmic Graphs

Describe how to transform the graph of $f(x) = \ln x$ into the graph of each function.

Graph each function.

c. $k(x) = \ln 3x$

Since $\ln 3x = \ln 3 + \ln x$, the graph of $k(x) = \ln 3x$ is obtained by translating the graph of $f(x) = \ln x$ up by $\ln 3 \approx 1.10$ units.



$[-2, 8]$ by $[-5, 5]$

Quick Review

Evaluate the expression without using a calculator.

1. $\log 10^3$

2. $\ln e^3$

3. $\log 10^{-2}$

Simplify the expression.

4. $\dfrac{x^3 y^{-3}}{x^{-2} y^2}$

5. $\dfrac{\left(x^2 y^4\right)^{1/2}}{2x^{-3}}$

Quick Review Solutions

Evaluate the expression without using a calculator.

1. $\log 10^3$ 3

2. $\ln e^3$ 3

3. $\log 10^{-2}$ -2

Simplify the expression.

4. $\dfrac{x^3y^{-3}}{x^{-2}y^2}$ $\dfrac{x^5}{y^5}$

5. $\dfrac{\left(x^2y^4\right)^{1/2}}{2x^{-3}}$ $\dfrac{x^4y^2}{2}$