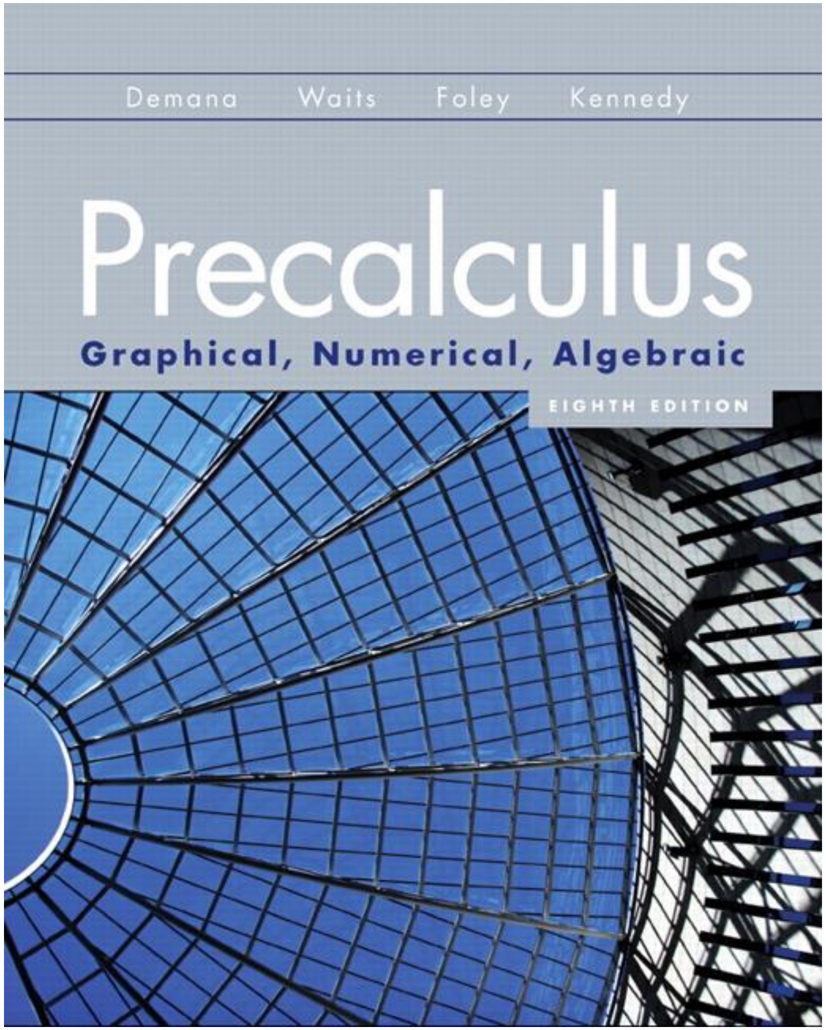
3.3

Logarithmic Functions and Their Graphs



What you'll learn about

- Inverses of Exponential Functions
- Common Logarithms – Base 10
- Natural Logarithms – Base e
- Graphs of Logarithmic Functions
- Measuring Sound Using Decibels

… and why

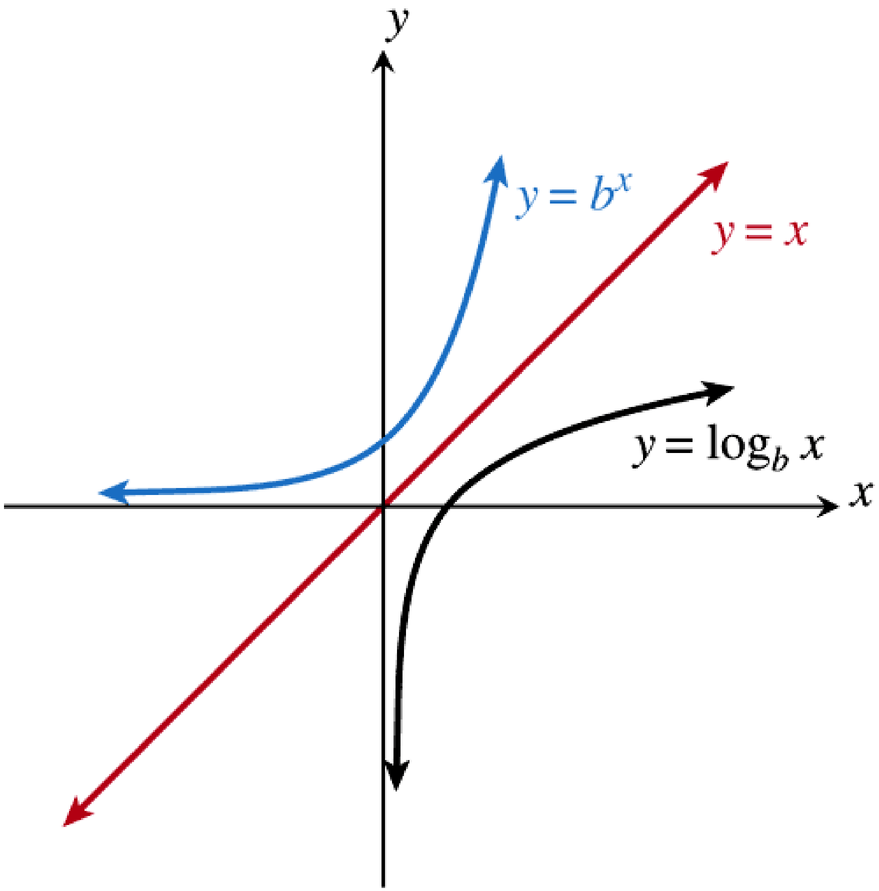
Logarithmic functions are used in many applications, including the measurement of the relative intensity of sounds.

Changing Between Logarithmic and Exponential Form

If $x > 0$ and $0 < b \neq 1$, then

$$y = \log_b(x) \text{ if and only if } b^y = x.$$

Inverses of Exponential Functions



Basic Properties of Logarithms

For $0 < b \neq 1,\ x > 0,$ and any real number y.

- $\log_b 1 = 0$ because $b^0 = 1$.
- $\log_b b = 1$ because $b^1 = b$.
- $\log_b b^y = y$ because $b^y = b^y$.
- $b^{\log_b x} = x$ because $\log_b x = \log_b x$.

An Exponential Function and Its Inverse

x	$f(x) = 2^x$
-3	$1/8$
-2	$1/4$
-1	$1/2$
0	1
1	2
2	4
3	8

x	$f^{-1}(x) = \log_2 x$
$1/8$	-3
$1/4$	-2
$1/2$	-1
1	0
2	1
4	2
8	3

Common Logarithm – Base 10

- Logarithms with base 10 are called common logarithms.
- The common logarithm $\log_{10} x = \log x$.
- The common logarithm is the inverse of the exponential function $y = 10^x$.

Basic Properties of Common Logarithms

Let x and y be real numbers with $x > 0$.

g $\log 1 = 0$ because $10^0 = 1$.

g $\log 10 = 1$ because $10^1 = 10$.

g $\log 10^y = y$ because $10^y = 10^y$.

g $10^{\log x} = x$ because $\log x = \log x$.

Example **Solving Simple Logarithmic Equations**

Solve the equation by changing it to exponential form.

$\log x = 4$

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Solve the equation by changing it to exponential form.

$\log x = 4$

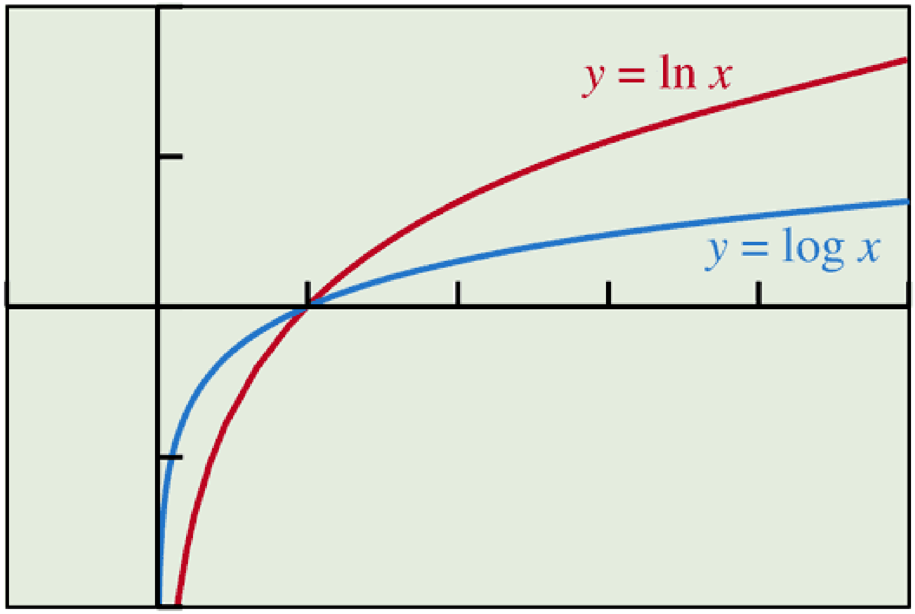
$$x = 10^4 = 10{,}000$$

Basic Properties of Natural Logarithms

Let x and y be real numbers with $x > 0$.

- $\ln 1 = 0$ because $e^0 = 1$.
- $\ln e = 1$ because $e^1 = e$.
- $\ln e^y = y$ because $e^y = e^y$.
- $e^{\ln x} = x$ because $\ln x = \ln x$.

Graphs of the Common and Natural Logarithm



[–1, 5] by [–2, 2]

Example Transforming Logarithmic Graphs

Describe transformations that will transform $f(x) = \ln x$ to $g(x) = 2 - 3\ln x$.

Example Transforming Logarithmic Graphs

Describe transformations that will transform $f(x) = \ln x$ to $g(x) = 2 - 3\ln x$.

Solve Algebraically

$$\begin{aligned} g(x) &= 2 - 3\ln x \\ &= -3\ln x + 2 \\ &= -f(x) + 2 \end{aligned}$$

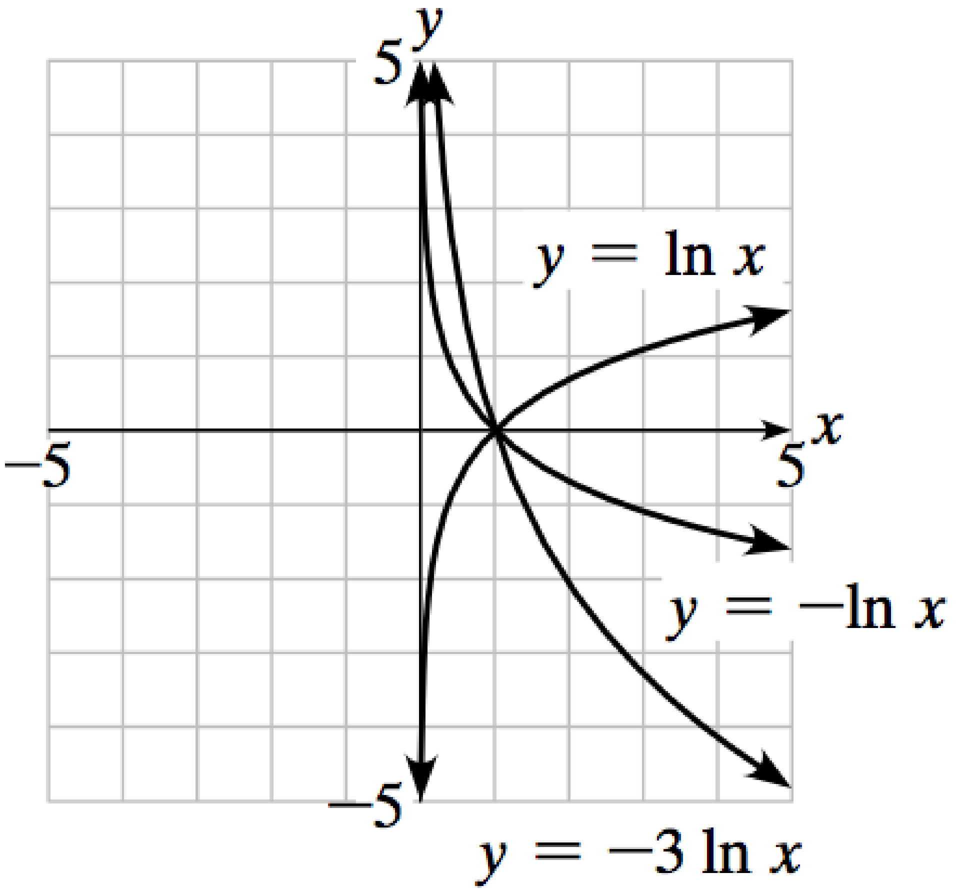
So g can be obtained by reflecting the graph of f across the x-axis, stretching vertically by a factor of 3, and then shifting upward 2 units.

Example Transforming Logarithmic Graphs

Describe transformations that will transform $f(x) = \ln x$ to $g(x) = 2 - 3\ln x$.

a. Reflect over the x-axis to obtain $y = -\ln x$

b. Stretch vertically by a factor of 3 to obtain $y = -3\ln x$



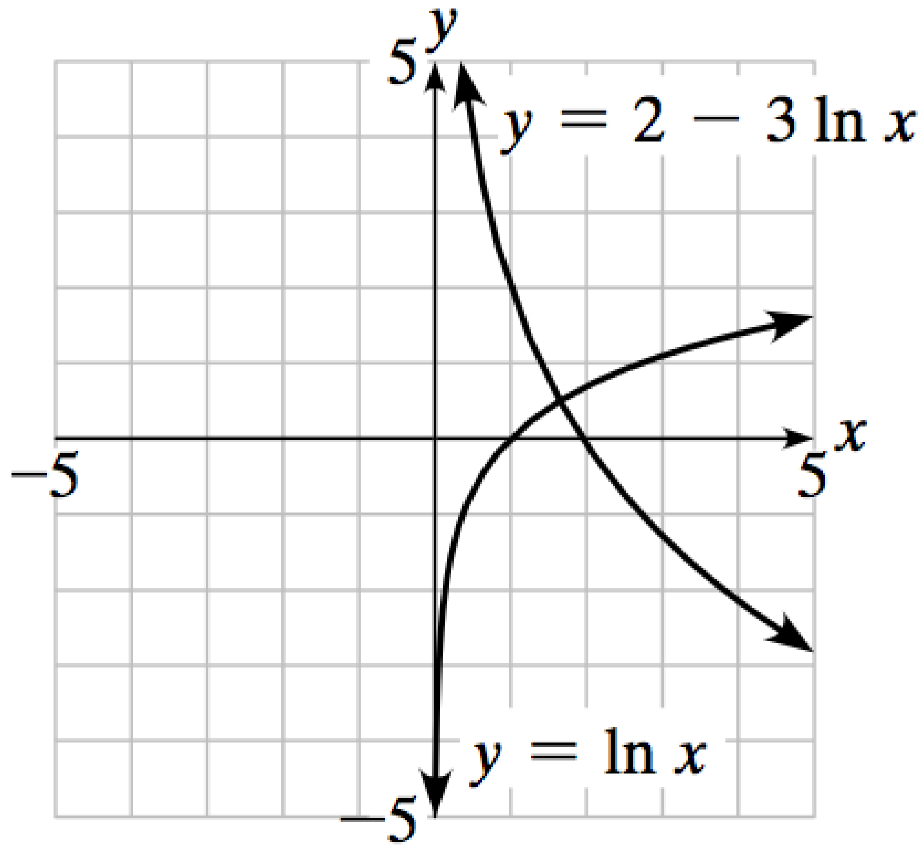
Example Transforming Logarithmic Graphs

Describe transformations that will transform $f(x) = \ln x$ to $g(x) = 2 - 3\ln x$.

c. Translate upward 2 units to obtain

$$y = -3\ln x + 2$$

$$y = 2 - 3\ln x$$



Decibels

The level of sound intensity in **decibels** (dB) is

$$\beta = 10\log\left(\frac{I}{I_0}\right),$$

where β (beta) is the number of decibels,

I is the sound intensity in W/m^2, and

$I_0 = 10^{-12}\ \text{W/m}^2$ is the threshold of human hearing (the quietest audible sound intensity).

Quick Review

Evaluate the expression without using a calculator.

1. 6^{-2}

2. $\dfrac{8^{11}}{2^{32}}$

3. 7^{0}

Rewrite as a base raised to a rational number exponent.

4. $\dfrac{1}{\sqrt{e^{3}}}$

5. $\sqrt[4]{10}$

Quick Review Solutions

Evaluate the expression without using a calculator.

1. 6^{-2} $\quad \dfrac{1}{36}$

2. $\dfrac{8^{11}}{2^{32}}$ $\quad 2$

3. 7^0 $\quad 1$

Rewrite as a base raised to a rational number exponent.

4. $\dfrac{1}{\sqrt{e^3}}$ $\quad e^{-3/2}$

5. $\sqrt[4]{10}$ $\quad 10^{1/4}$