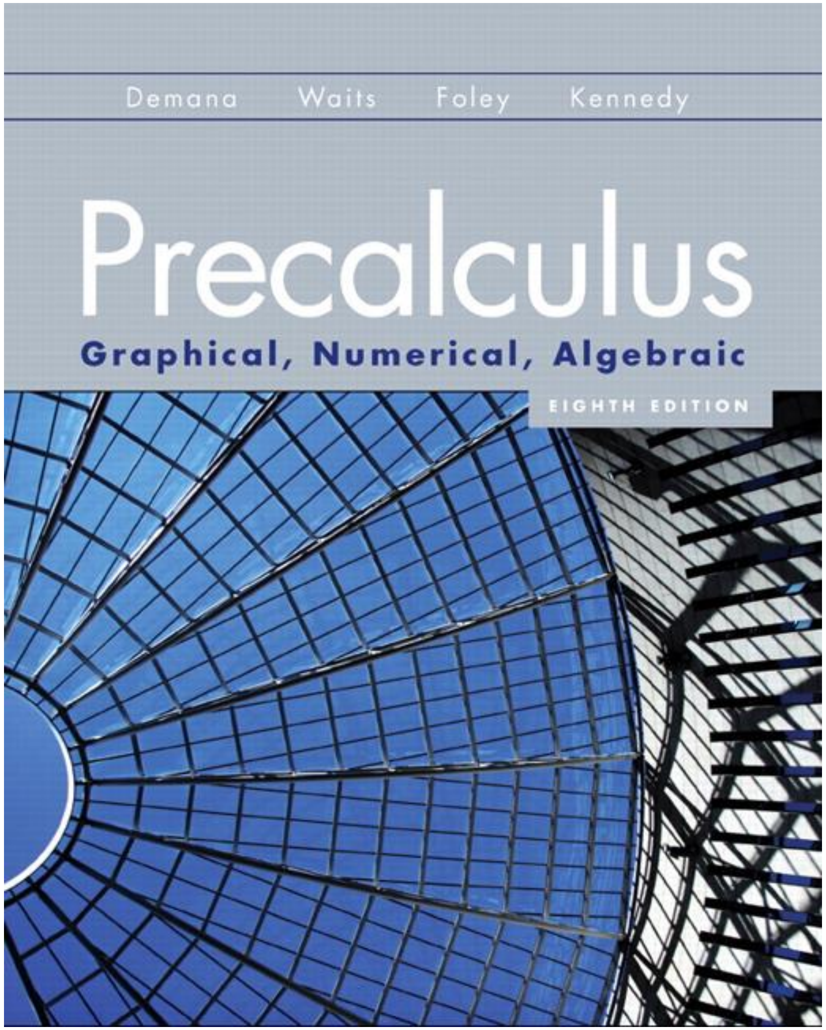
2.8

Solving Inequalities in One Variable



What you'll learn about

- Polynomial Inequalities
- Rational Inequalities
- Other Inequalities
- Applications

… and why

Designing containers as well as other types of applications often require that an inequality be solved.

Polynomial Inequalities

A polynomial inequality takes the form $f(x) > 0$, $f(x) \geq 0$, $f(x) < 0$, $f(x) \leq 0$ or $f(x) \neq 0$, where $f(x)$ is a polynomial.

- To solve $f(x) > 0$ is to find the values of x that make $f(x)$ positive.
- To solve $f(x) < 0$ is to find the values of x that make $f(x)$ negative.

Example **Finding where a Polynomial is Zero, Positive, or Negative**

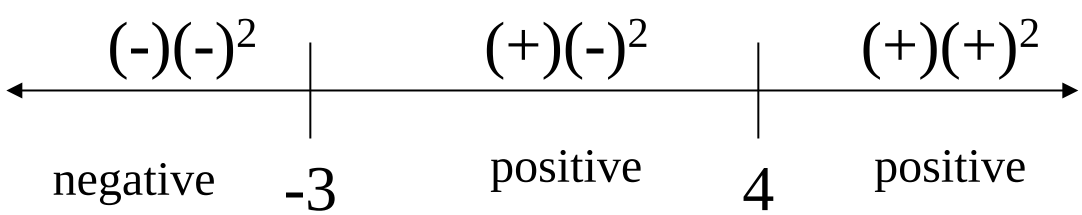
Let $f(x) = (x+3)(x-4)^2$. Determine the real number values of x that cause $f(x)$ to be (a) zero, (b) positive, (c) negative.

Example **Finding where a Polynomial is Zero, Positive, or Negative**

Let $f(x) = (x+3)(x-4)^2$. Determine the real number values of x that cause $f(x)$ to be (a) zero, (b) positive, (c) negative.

(a) The real zeros are at $x = -3$ and at $x = 4$ (multiplicity 2).

Use a sign chart to find the intervals when $f(x) > 0$, $f(x) < 0$.



(b) $f(x) > 0$ on the interval $(-3, 4) \cup (4, \infty)$.

(c) $f(x) < 0$ on the interval $(-\infty, -3)$.

Example **Finding where a Polynomial is Zero, Positive, or Negative**

Let $r(x) = \dfrac{(x+3)(x-5)}{(5x-2)}$.

Determine the real number values of x that cause $r(x)$ to be (a) zero, (b) undefined (c) positive, (d) negative.

Example **Finding where a Polynomial is Zero, Positive, or Negative**

Let $r(x)=\dfrac{(x+3)(x-5)}{(5x-2)}$.

(a) $r(x)=0$ when its numerator is 0.

$$\left(x+3\right)\left(x-5\right)=0 \Leftrightarrow x=-3 \text{ or } x=5$$

$$r\left(x\right)=0 \text{ when } x=-3 \text{ or } x=5$$

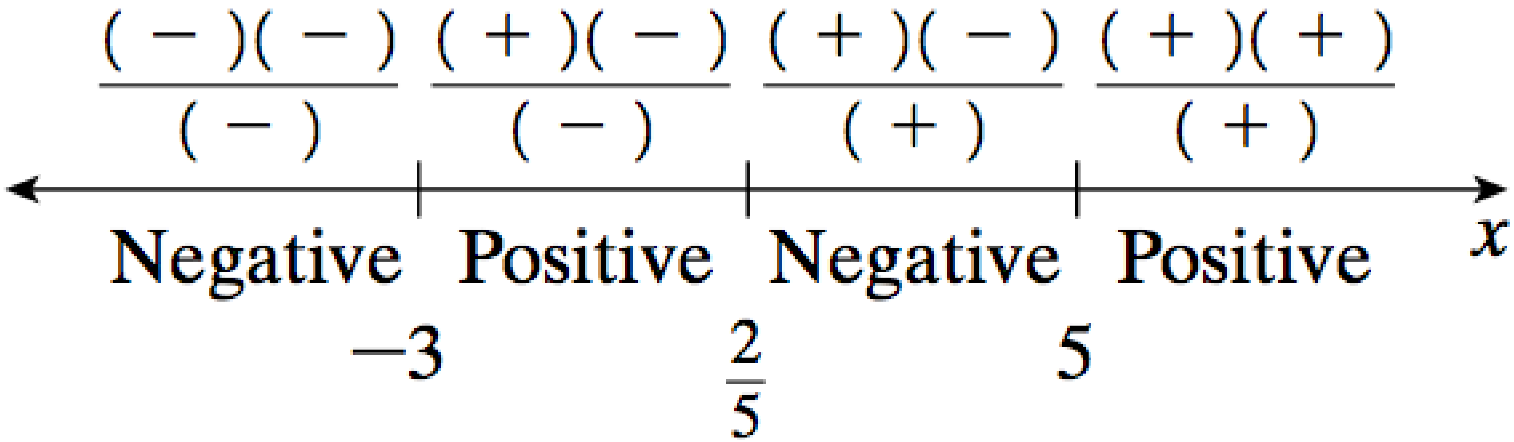
(b) $r(x)$ is undefined when its denominator is 0.

$$5x-2=0 \Leftrightarrow x=\frac{2}{5}$$

$$r\left(x\right) \text{ is undefined when } x=\frac{2}{5}$$

Example **Finding where a Polynomial is Zero, Positive, or Negative**

Make a sign chart.



(c) $r(x)$ is positive if $-3 < x < \frac{2}{5}$ or $x > 5$

(d) $r(x)$ is negative if $x < -3\frac{2}{5} <$ or $x < 5$

Example **Solving a Polynomial Inequality Graphically**

Solve $x^3 - 6x^2 \le 2 - 8x$ graphically.

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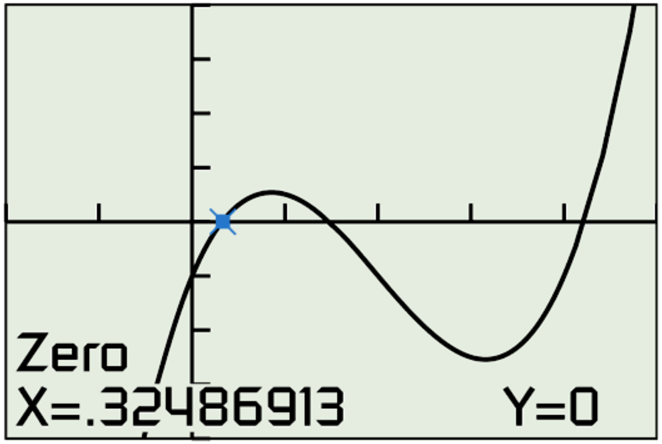
Solve $x^3 - 6x^2 \le 2 - 8x$ graphically.

Rewrite the inequality $x^3 - 6x^2 + 8x - 2 \le 0.$

Let $f(x) = x^3 - 6x^2 + 8x - 2$ and find the real zeros of f graphically.



[–2, 5] by [–8, 8]

The three real zeros are approximately 0.32, 1.46, and 4.21. The solution consists of the x values for which the graph is on or below the x-axis.

The solution is $(-\infty, 0.32] \cup [1.46, 4.21]$.

Example **Creating a Sign Chart for a Rational Function**

Let $r(x)=\dfrac{x+1}{(x+3)(x-1)}$.

Determine the values of x that cause $r(x)$ to be

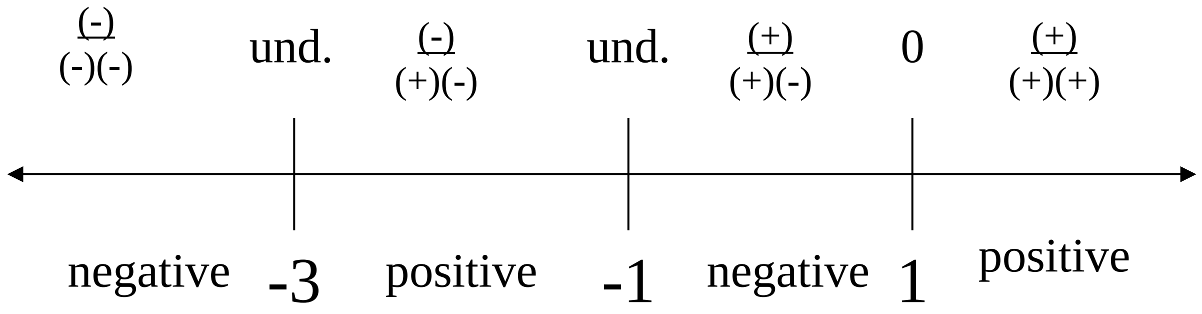
(a) zero, (b) undefined, (c) positive, and (d) negative.

Example **Creating a Sign Chart for a Rational Function**

Let $r(x)=\dfrac{x+1}{(x+3)(x-1)}$.

(a) $r(x)=0$ when $x=-1$.

(b) $r(x)$ is undefined when $x=-3$ and $x=1$.



(c) $(-3,-1)\cup(1,\infty)$

(d) $(-\infty,-3)\cup(-1,1)$

Example **Solving an Inequality Involving a Radical**

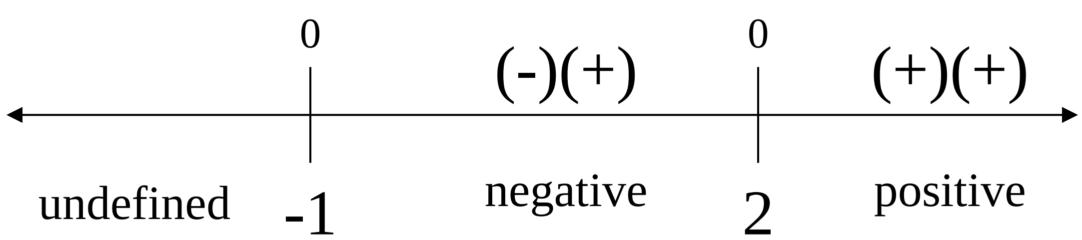
Solve $(x-2)\sqrt{x+1} \le 0.$

Example **Solving an Inequality Involving a Radical**

Solve $(x-2)\sqrt{x+1} \le 0.$

Let $f(x) = (x-2)\sqrt{x+1}$. Because of the factor $\sqrt{x+1}$, $f(x)$ is undefined if $x < -1$.

The zeros are at $x = -1$ and $x = 2$.



$f(x) \le 0$ over the interval $[-1,2]$.

Quick Review

Use limits to state the end behavior of the function.

1. $f(x) = 2x^3 - 2x + 5$

2. $g(x) = -2x^4 + 2x^2 - x + 1$

Combine the fractions, reduce your answer to lowest terms.

3. $\dfrac{2}{x^2} + x$

4. $x^2 + \dfrac{1}{x}$

List all the possible rational zeros and facotr completely.

5. $x^3 + x^2 - 4x - 4$

Quick Review Solutions

Use limits to state the end behavior of the function.

1. $f(x) = 2x^3 - 2x + 5$ $\lim_{x \to -\infty} f(x) = -\infty$ $\lim_{x \to \infty} f(x) = \infty$

2. $g(x) = -2x^4 + 2x^2 - x + 1$ $\lim_{x \to -\infty} g(x) = \lim_{x \to \infty} g(x) = -\infty$

Combine the fractions, reduce your answer to lowest terms.

3. $\dfrac{2}{x^2} + x$ $\dfrac{2 + x^3}{x^2}$

4. $x^2 + \dfrac{1}{x}$ $\dfrac{x^3 + 1}{x}$

List all the possible rational zeros and facotr completely.

5. $x^3 + x^2 - 4x - 4$ $\pm 4, \pm 2, \pm 1;$ $(x+2)(x-2)(x+1)$

Chapter Test

1. Write an equation for the linear function f satisfying the given condition: $f(-3) = -2$ and $f(4) = -9$.

2. Write an equation for the quadratic function whose graph contains the vertex $(-2, -3)$ and the point $(1, 2)$.

3. Write the statement as a power function equation. Let k be the constant of variation. The surface area S of a sphere varies directly as the square of the radius r.

Chapter Test

4. Divide $f(x)$ by $d(x)$, and write a summary statement in polynomial form: $f(x) = 2x^3 - 7x^2 + 4x - 5;\ d(x) = x - 3$

5. Use the Rational Zeros Theorem to write a list of all potential rational zeros. Then determine which ones, if any, are zeros. $f(x) = 2x^4 - x^3 - 4x^2 - x - 6$

6. Find all zeros of the function. $f(x) = x^4 - 10x^3 + 23x^2$

Chapter Test

7. Find all zeros and write a linear factorization of the function. $f(x) = 5x^3 - 24x^2 + x + 12$

8. Find the asymptotes and intercepts of the function.

$$f(x) = \frac{x^2 + x + 1}{x^2 - 1}$$

9. Solve the equation or inequality algebraically.

$$2x + \frac{12}{x} = 11$$

Chapter Test

10. Larry uses a slingshot to launch a rock straight up from a point 6 ft above level ground with an initial velocity of 170 ft/sec.

(a) Find an equation that models the height of the rock t seconds after it is launched.

(b) What is the maximum height of the rock?

(c) When will it reach that height?

(d) When will the rock hit the ground?

Chapter Test Solutions

1. Write an equation for the linear function f satisfying the given condition: $f(-3) = -2$ and $f(4) = -9$.

$$y = -x - 5$$

2. Write an equation for the quadratic function whose graph contains the vertex $(-2, -3)$ and the point $(1, 2)$.

$$y = \frac{5}{9}(x+2)^2 - 3$$

3. Write the statement as a power function equation. Let k be the constant of variation. The surface area S of a sphere varies directly as the square of the radius r.

$$s = kr^2$$

Chapter Test Solutions

4. Divide $f(x)$ by $d(x)$, and write a summary statement in polynomial form: $f(x) = 2x^3 - 7x^2 + 4x - 5;\ \ d(x) = x - 3$

$$2x^2 - x + 1 - \frac{2}{x-3}$$

5. Use the Rational Zeros Theorem to write a list of all potential rational zeros. Then determine which ones, if any, are zeros. $f(x) = 2x^4 - x^3 - 4x^2 - x - 6$

$$\pm 1, \pm 2, \pm 3, \pm 6, \pm 1/2, \pm 3/2; \quad -3/2 \text{ and } 2$$

6. Find all zeros of the function. $f(x) = x^4 - 10x^3 + 23x^2$

$$0,\ 5 \pm \sqrt{2}$$

Chapter Test Solutions

7. Find all zeros and write a linear factorization of the function. $f(x) = 5x^3 - 24x^2 + x + 12$

$$4/5,\ 2 \pm \sqrt{7}$$

8. Find the asymptotes and intercepts of the function.

$$f(x) = \frac{x^2 + x + 1}{x^2 - 1}$$

y-intercept $(0,1)$, x-intercept none, VA: $x = -1$ HA: $y = 1$

9. Solve the equation or inequality algebraically.

$$2x + \frac{12}{x} = 11$$

$x = 3/2$ or $x = 4$

Chapter Test Solutions

10. Larry uses a slingshot to launch a rock straight up from a point 6 ft above level ground with an initial velocity of 170 ft/sec. $h = -16t^2 + 170t + 6$

(a) Find an equation that models the height of the rock t seconds after it is launched. $h = -16t^2 + 170t + 6$

(b) What is the maximum height of the rock? 457.562 ft

(c) When will it reach that height? 5.3125 sec

(d) When will the rock hit the ground? 10.66 sec