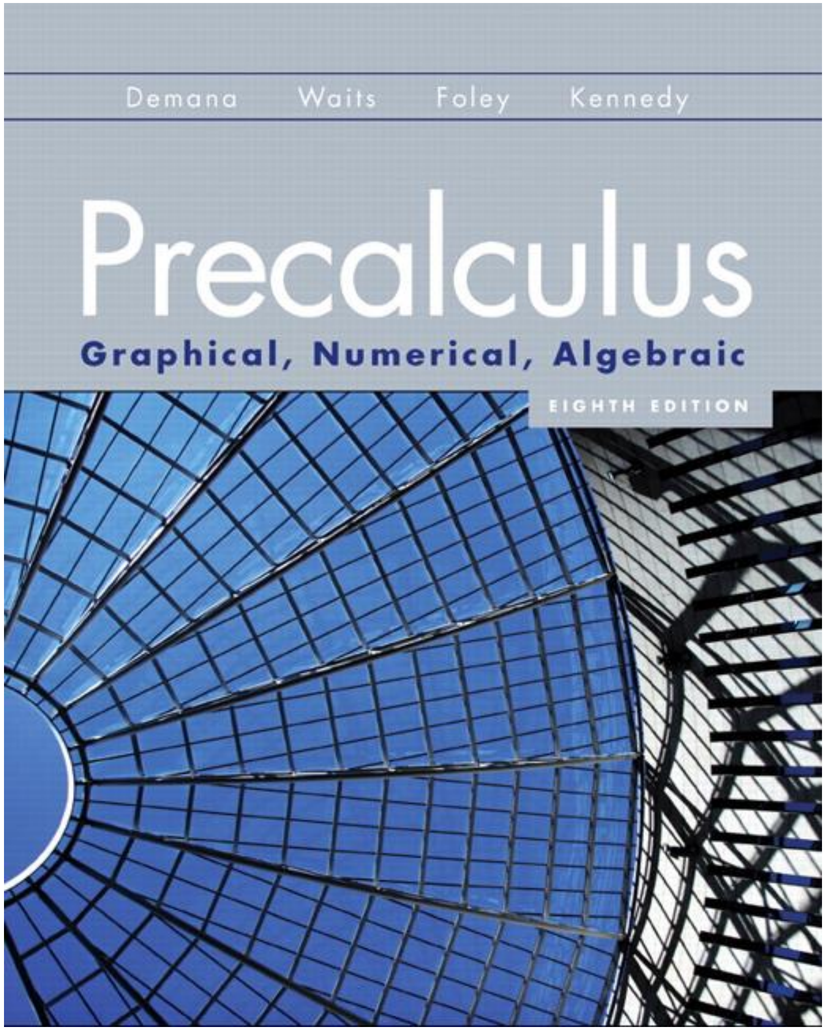
2.7

Solving Equations in One Variable



What you'll learn about

- Solving Rational Equations
- Extraneous Solutions
- Applications

… and why

Applications involving rational functions as models often require that an equation involving fractions be solved.

Extraneous Solutions

When we multiply or divide an equation by an expression containing variables, the resulting equation may have solutions that are *not* solutions of the original equation. These are **extraneous solutions**. For this reason we must check each solution of the resulting equation in the original equation.

Example **Solving by Clearing Fractions**

Solve $x + \dfrac{1}{x-3} = 1.$

Example Solving by Clearing Fractions

Solve $x + \dfrac{1}{x-3} = 1.$

The LCD is $x - 3$.

$$x + \frac{1}{x-3} = 1$$
$$x(x-3) + 1 = x - 3$$
$$x^2 - 3x + 1 = x - 3$$
$$x^2 - 4x + 4 = 0$$
$$(x-2)^2 = 0$$
$$x = 2$$

Support numerically:

For $x = 2: \quad x + \dfrac{1}{x-3}$

$$= 2 + \frac{1}{2-3} = 2 + \frac{1}{-1} = 1.$$

$x = 2$ is the solution of the original equation.

Example **Solving by Clearing Fractions**

Solve $x + \dfrac{2}{x} = 3.$

Example **Solving by Clearing Fractions**

Solve $x + \dfrac{2}{x} = 3.$

The LCD is x.

$$x + \frac{2}{x} = 3$$

$$x^2 + 2 = 3x \qquad \text{multiply by } x$$

$$x^2 - 3x + 2 = 0 \qquad \text{subtract } 3x$$

$$(x-2)(x-1) = 0 \qquad \text{factor}$$

$$x = 2 \quad \text{or} \quad x = 1$$

Confirm algebraically:

Let $x = 2$: $\quad 2 + \dfrac{2}{2} = 3$

Let $x = 1$: $\quad 1 + \dfrac{2}{1} = 3$

Each value is a solution of the original equation.

Example **Eliminating Extraneous Solutions**

Solve the equation $\dfrac{1}{x-3}+\dfrac{2x}{x-1}=\dfrac{2}{x^2-4x+3}$.

Example Eliminating Extraneous Solutions

The LCD is $(x-1)(x-3)$.

$$(x-1)(x-3)\left(\frac{1}{x-3}+\frac{2x}{x-1}\right)=(x-1)(x-3)\left(\frac{2}{x^2-4x+3}\right)$$

$$(x-1)(1)+2x(x-3)=2$$

$$2x^2-5x-3=0$$

$$(2x+1)(x-3)=0$$

$$x=-1/2 \quad \text{or} \quad x=3$$

Check solutions in the original equation. $x=-1/2$ is the only solution. The original equation is not defined at $x=3$.

Example **Finding a Minimum Perimeter**

Find the dimensions of the rectangle with minimum perimeter if its area is 300 square meters.

Find this least perimeter.

Example **Finding a Minimum Perimeter**

Word Statement: Perimeter $= 2 \times$ length $+ 2 \times$ width

$x =$ width in meters $\quad 300 / x =$ length in meters

Function to be minimized: $P(x) = 2x + 2\left(\dfrac{300}{x}\right) = 2x + \dfrac{600}{x}$

Solve graphically: A minimum of approximately 69.28 occurs when $x \approx 17.32$

The width is 17.32 m and the length is 300/17.32=17.32 m.

The minimum perimeter is 69.28 m.

Quick Review

Find the missing numerator or denominator.

1. $\dfrac{2}{x+3} = \dfrac{?}{x^2+2x-3}$

2. $\dfrac{x-4}{x+4} = \dfrac{x^2-16}{?}$

Find the LCD and rewrite the expression as a single fraction reduced to lowest terms.

3. $\dfrac{3}{2} + \dfrac{5}{4} - \dfrac{7}{12}$

4. $\dfrac{x}{x-1} + \dfrac{2}{x}$

Use the quadratic formula to find the zeros of the quadratic polynomial.

5. $2x^2 + 4x - 1$

Quick Review Solutions

Find the missing numerator or denominator.

1. $\dfrac{2}{x+3} = \dfrac{?}{x^2+2x-3}$: $2x-2$ 2. $\dfrac{x-4}{x+4} = \dfrac{x^2-16}{?}$: $x^2+8x+16$

Find the LCD and rewrite the expression as a single fraction reduced to lowest terms.

3. $\dfrac{3}{2}+\dfrac{5}{4}-\dfrac{7}{12}=\dfrac{13}{6}$

4. $\dfrac{x}{x-1}+\dfrac{2}{x}=\dfrac{x^2+2x-2}{x^2-x}$

Use the quadratic formula to find the zeros of the quadratic polynomial.

5. $2x^2+4x-1$ $\quad x=\dfrac{-2\pm\sqrt{6}}{2}$