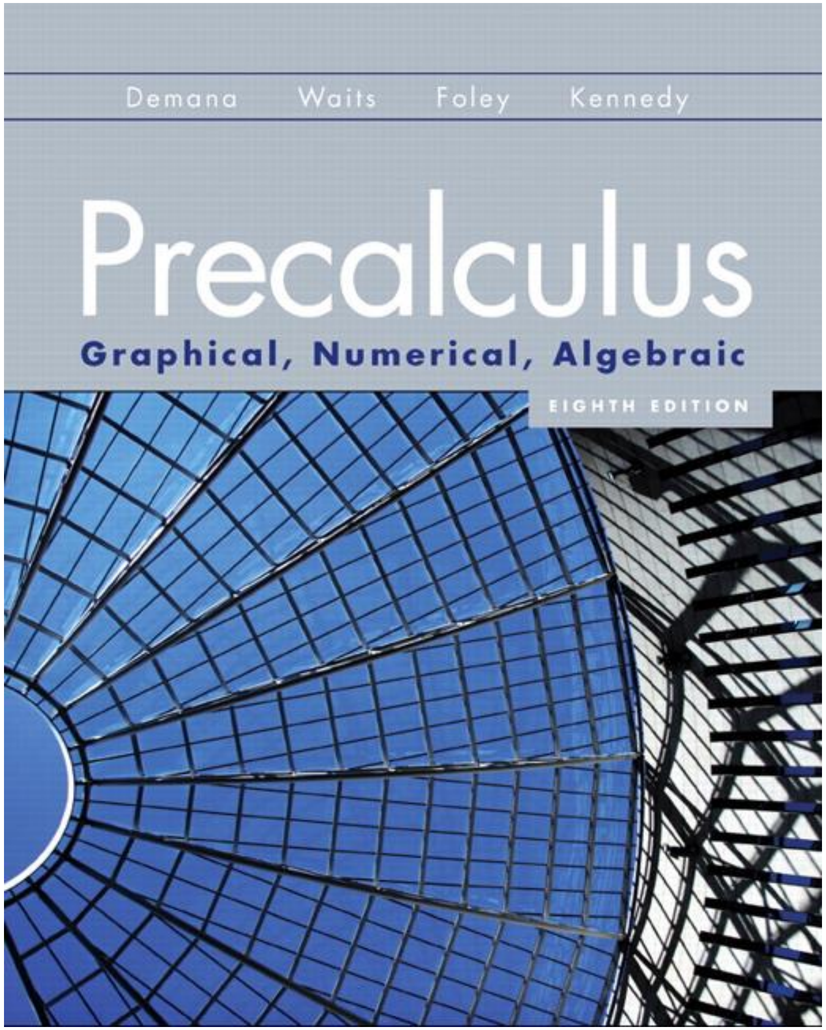
2.3
Polynomial Functions of Higher Degree with Modeling



What you'll learn about

- Graphs of Polynomial Functions
- End Behavior of Polynomial Functions
- Zeros of Polynomial Functions
- Intermediate Value Theorem
- Modeling

… and why

These topics are important in modeling and can be used to provide approximations to more complicated functions, as you will see if you study calculus.

The Vocabulary of Polynomials

- Each monomial in the sum – $a_n x^n, a_{n-1} x^{n-1}, \ldots, a_0$ – is a **term** of the polynomial.
- A polynomial function written in this way, with terms in descending degree, is written in **standard form**.
- The constants $a_n, a_{n-1}, \ldots, a_0$ – are the **coefficients** of the polynomial.
- The term $a_n x^n$ is the **leading term**, and a_0 is the constant term.

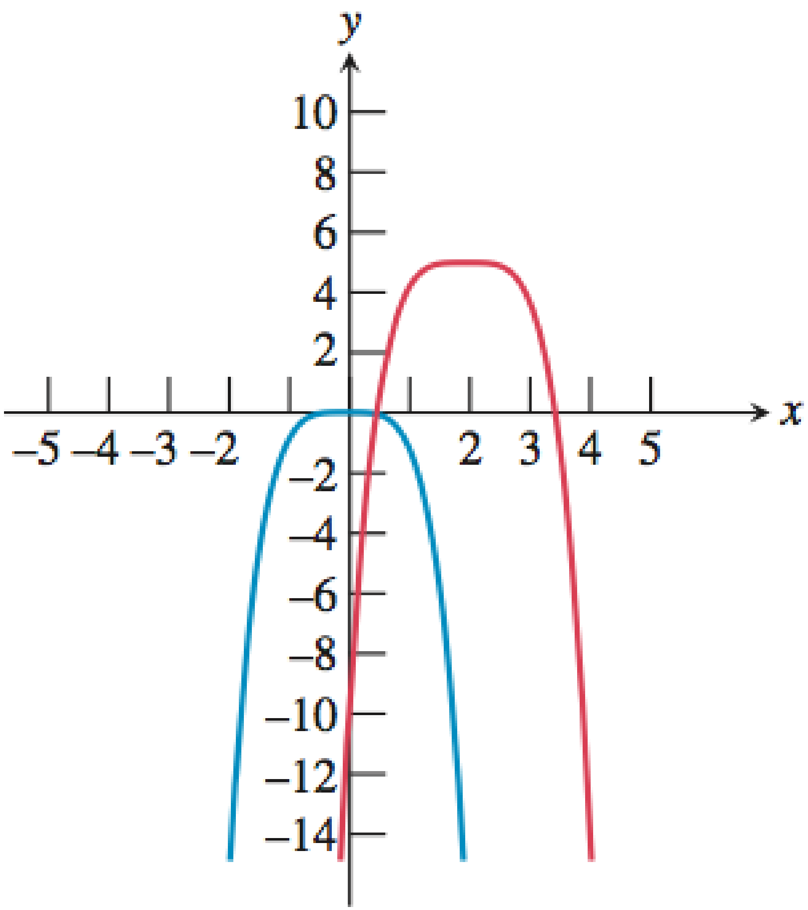
Example **Graphing Transformations of Monomial Functions**

Describe how to transform the graph of an appropriate monomial function $f(x) = a_n x^n$ into the graph of $h(x) = -(x-2)^4 + 5.$

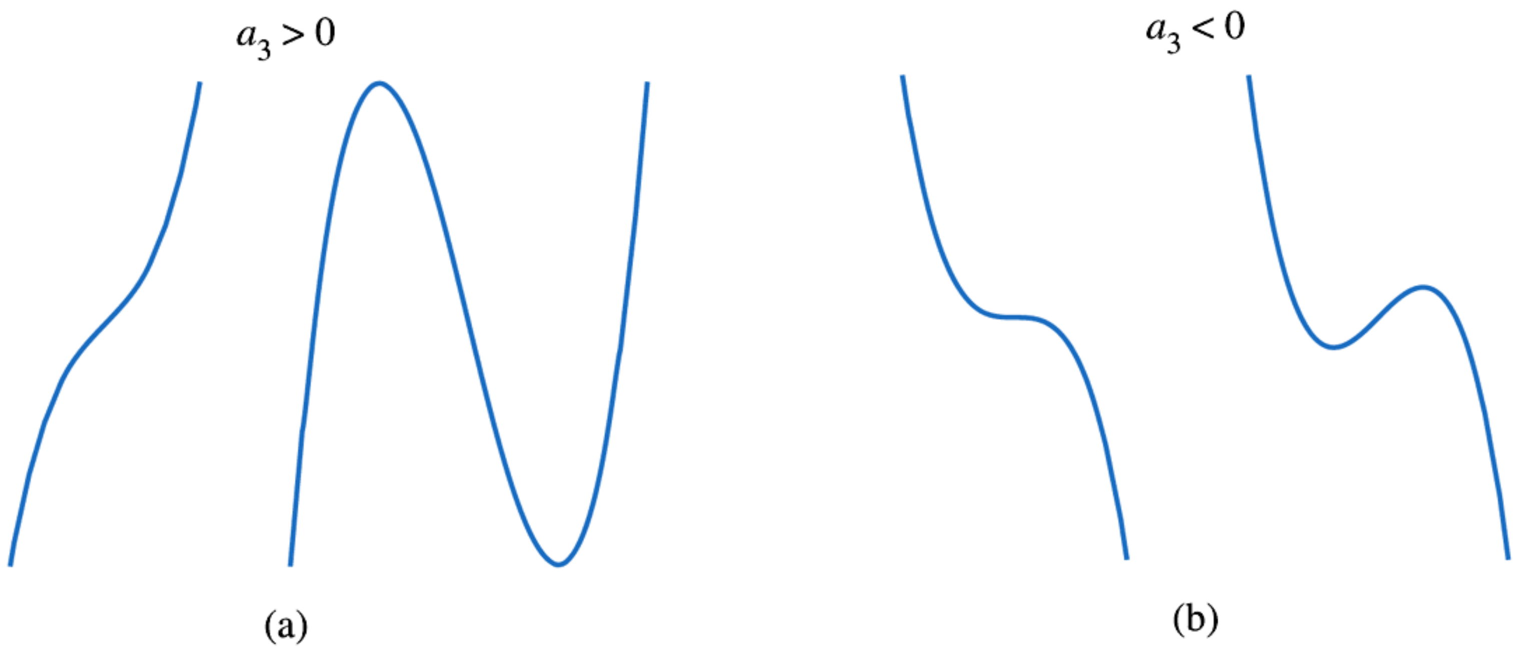
Sketch $h(x)$ and compute the y-intercept.

Example Graphing Transformations of Monomial Functions

You can obtain the graph of $h(x) = -(x-2)^4 + 5$ by shifting the graph of $f(x) = -x^4$ two units to the right and five units up. The y-intercept of $h(x)$ is $h(0) = -(-2)^4 + 5 = -11.$



Cubic Functions



Quartic Function

(a) (b)

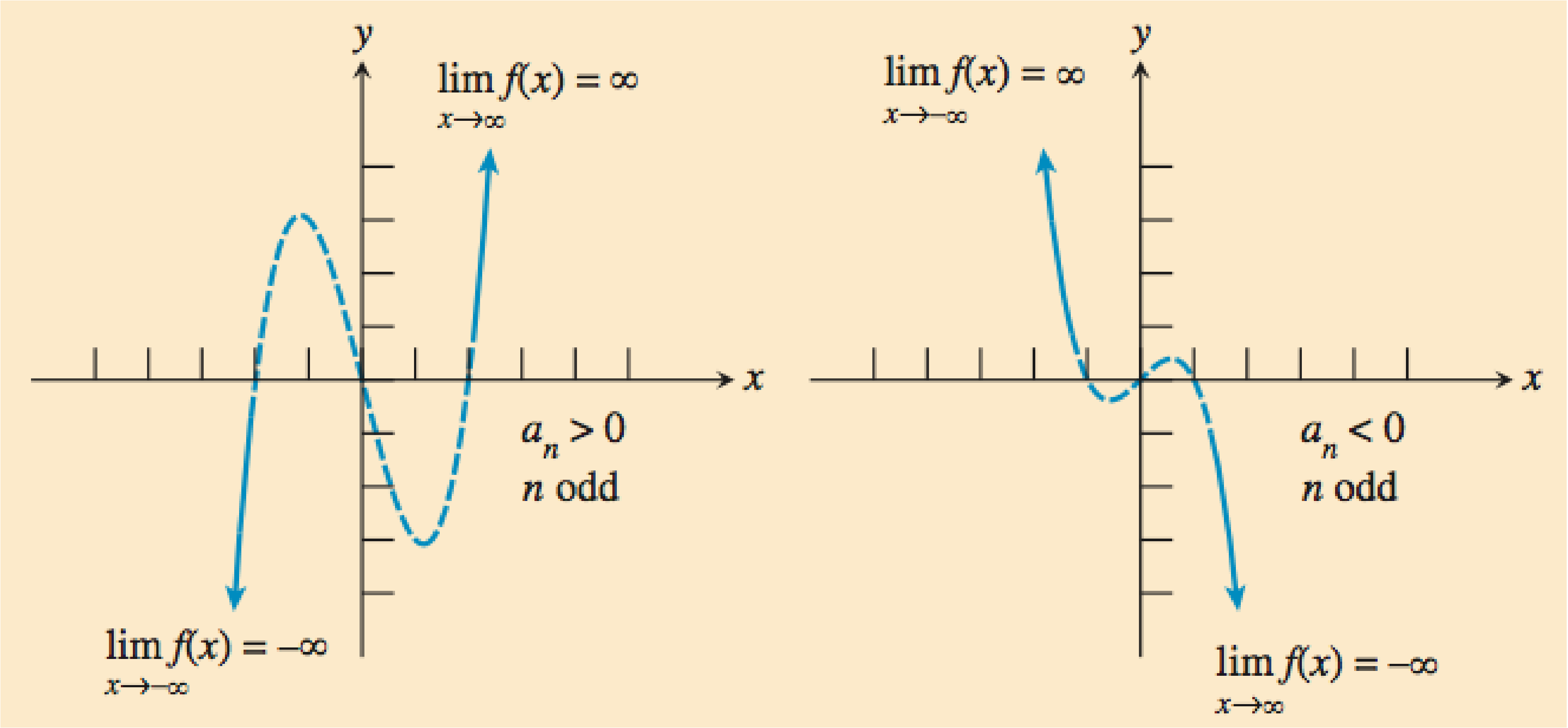
Local Extrema and Zeros of Polynomial Functions

A polynomial function of degree n has at most $n - 1$ local extrema and at most n zeros.

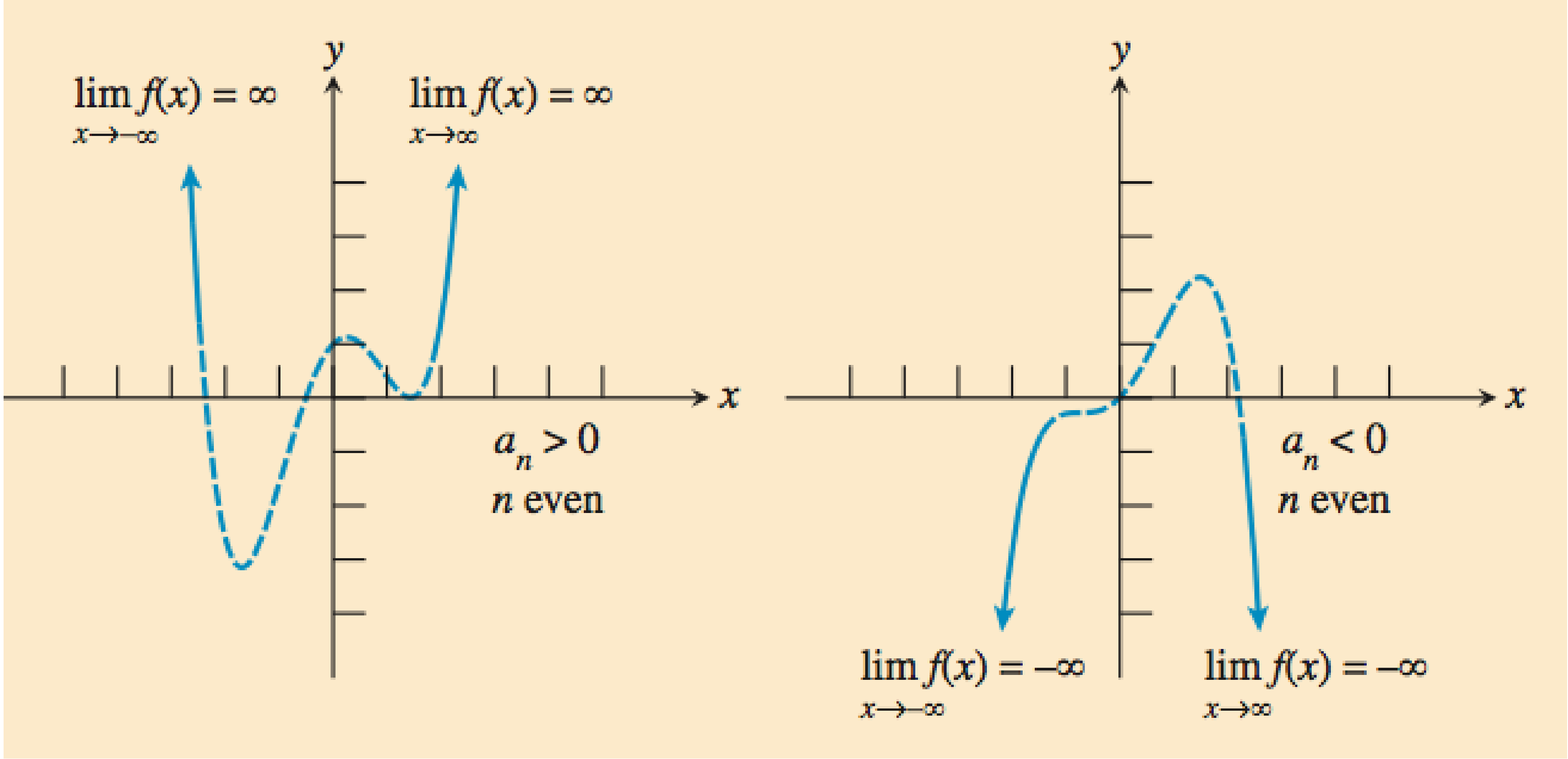
Leading Term Test for Polynomial End Behavior

For any polynomial function $f(x) =$ [illegible] $+ \cdots + a_1 x + a_0$, the limits $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ are determined by the degree [illegible] of the polynomial and its leading coefficient [illegible]:

Leading Term Test for Polynomial End Behavior



Leading Term Test for Polynomial End Behavior



Example **Applying Polynomial Theory**

Describe the end behavior of $g(x) = 2x^4 - 3x^3 + x - 1$ using limits.

Example Applying Polynomial Theory

Describe the end behavior of $g(x) = 2x^4 - 3x^3 + x - 1$ using limits.

$$\lim_{x \to \pm\infty} g(x) = \infty$$

Example **Finding the Zeros of a Polynomial Function**

Find the zeros of $f(x) = 2x^3 - 4x^2 - 6x$.

Example **Finding the Zeros of a Polynomial Function**

Find the zeros of $f(x) = 2x^3 - 4x^2 - 6x$.

$$\text{Solve } f(x) = 0$$

$$2x^3 - 4x^2 - 6x = 0$$

$$2x\left(x+1\right)\left(x-3\right) = 0$$

$$x = 0, x = -1, x = 3$$

Multiplicity of a Zero of a Polynomial Function

If f is a polynomial function and $(x-c)^m$ is a factor of f but $(x-c)^{m+1}$ is not, then c is a zero of **multiplicity** $\boldsymbol{m}$ of f.

Zeros of Odd and Even Multiplicity

If a polynomial function f has a real zero c of odd multiplicity, then the graph of f crosses the x-axis at $(c, 0)$ and the value of f changes sign at $x = c$. If a polynomial function f has a real zero c of even multiplicity, then the graph of f does not cross the x-axis at $(c, 0)$ and the value of f does not change sign at $x = c$.

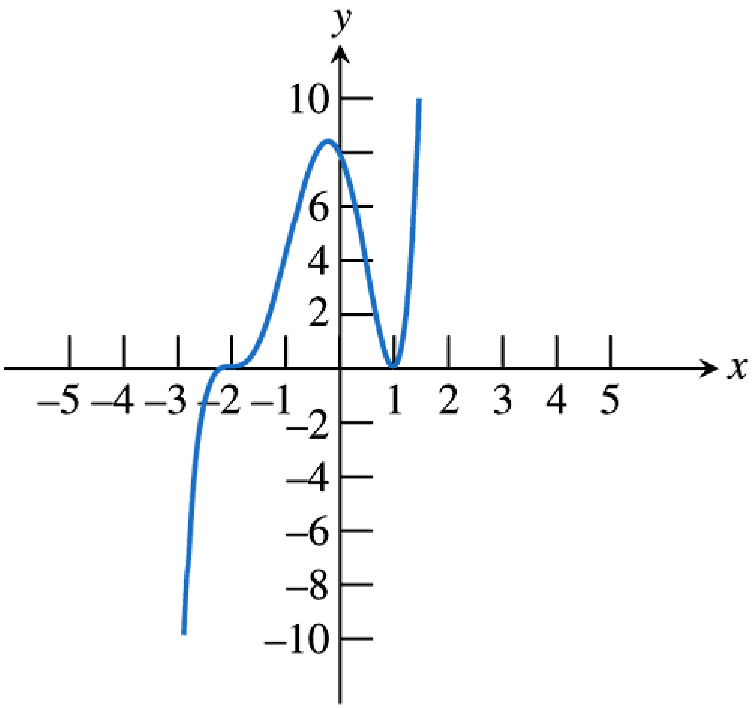
Example **Sketching the Graph of a Factored Polynomial**

Sketch the graph of $f(x) = (x+2)^3(x-1)^2$.

Example **Sketching the Graph of a Factored Polynomial**

Sketch the graph of $f(x) = (x+2)^3(x-1)^2$.

The zeros are $x = -2$ and $x = 1$. The graph crosses the x-axis at $x = -2$ because the multiplicity 3 is odd. The graph does not cross the x-axis at $x = 1$ because the multiplicity 2 is even.

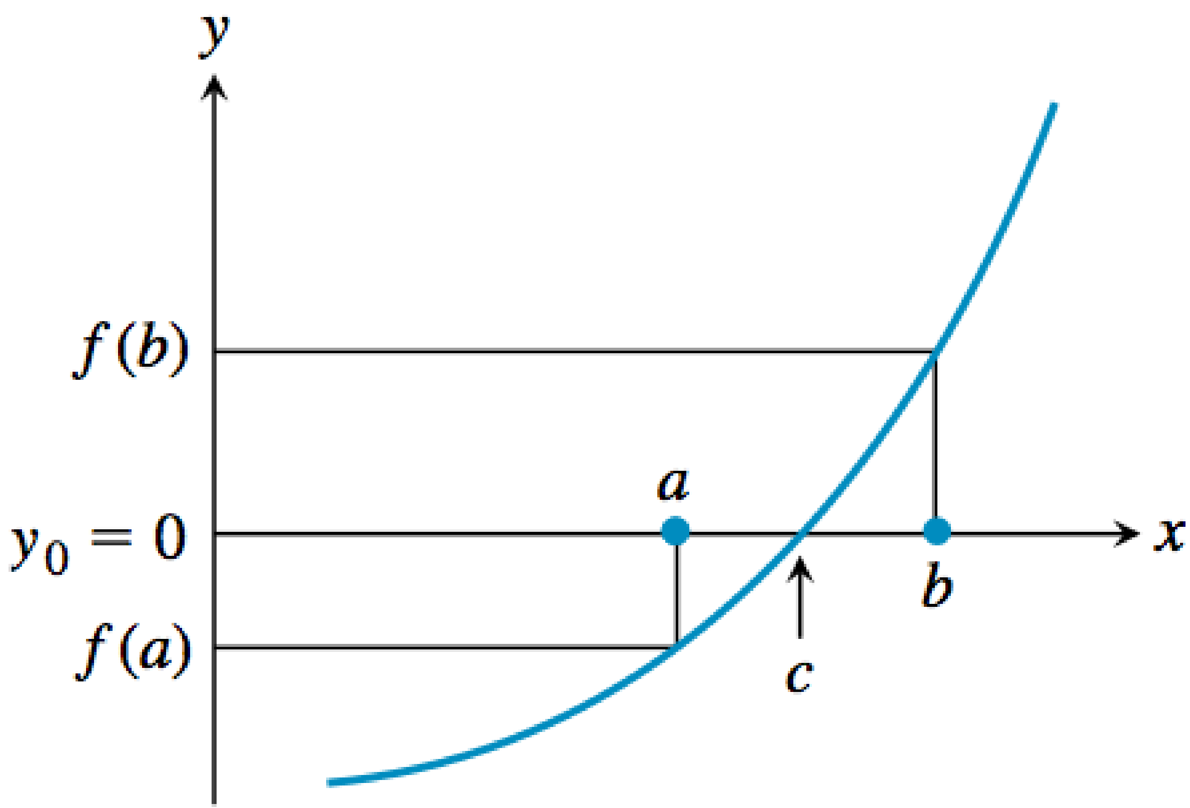


Intermediate Value Theorem

If a and b are real numbers with $a < b$ and if f is continuous on the interval $[a,b]$, then f takes on every value between $f(a)$ and $f(b)$. In other words, if y_0 is between $f(a)$ and $f(b)$, then $y_0=f(c)$ for some number c in $[a,b]$.

In particular, if $f(a)$ and $f(b)$ have opposite signs (i.e., one is negative and the other is positive, then $f(c) = 0$ for some number c in $[a, b]$.

Intermediate Value Theorem



Quick Review

Factor the polynomial into linear factors.

1. $3x^2 - 11x - 4$

2. $4x^3 + 10x^2 - 24x$

Solve the equation mentally.

3. $x(x-2) = 0$

4. $2(x+2)^2(x+1) = 0$

5. $x^3(x+3)(x-5) = 0$

Quick Review Solutions

Factor the polynomial into linear factors.

1. $3x^2 - 11x - 4$ $\quad (3x+1)(x-4)$

2. $4x^3 + 10x^2 - 24x$ $\quad 2x(2x-3)(x+4)$

Solve the equation mentally.

3. $x(x-2) = 0$ $\quad x = 0, x = 2$

4. $2(x+2)^2(x+1) = 0$ $\quad x = -2, x = -1$

5. $x^3(x+3)(x-5) = 0$ $\quad x = 0, x = -3, x = 5$