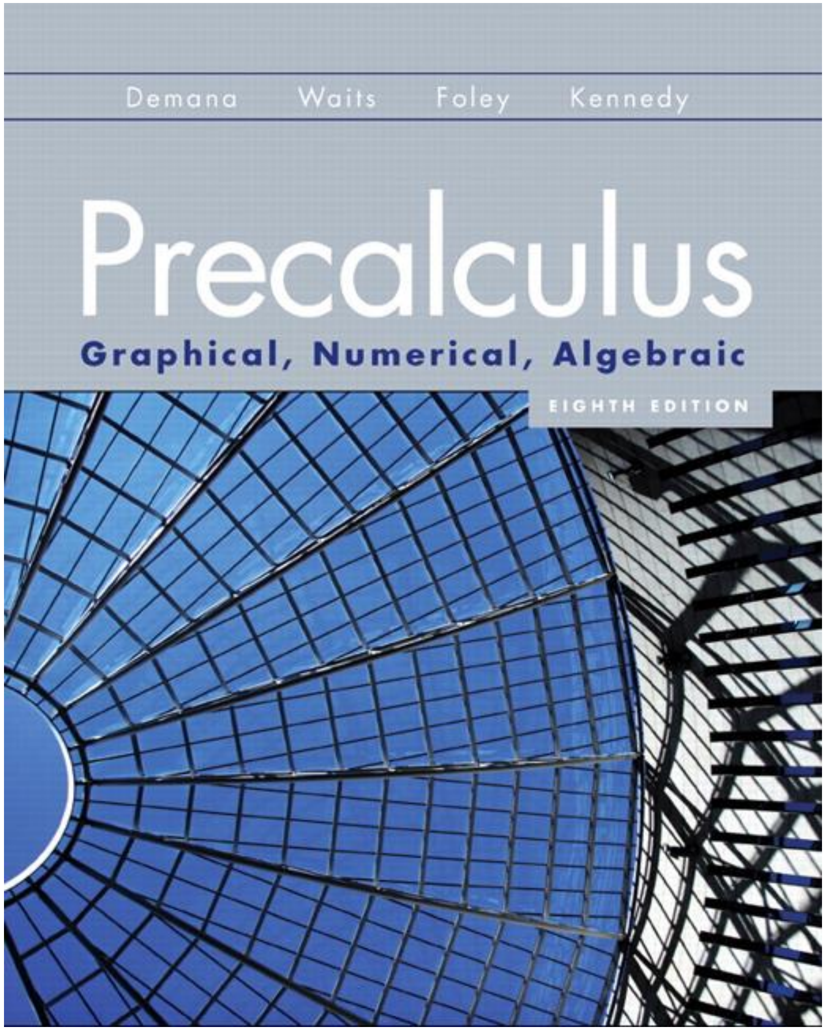
2.1

Linear and Quadratic Functions and Modeling



PEARSON
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What you'll learn about

- Polynomial Functions
- Linear Functions and Their Graphs
- Average Rate of Change
- Linear Correlation and Modeling
- Quadratic Functions and Their Graphs
- Applications of Quadratic Functions

… and why

Many business and economic problems are modeled by linear functions. Quadratic and higher degree polynomial functions are used to model some manufacturing applications.

Polynomial Function

Let n be a nonnegative integer and let $a_0, a_1, a_2, \ldots, a_{n-1}, a_n$ be real numbers with $a_n \neq 0$. The function given by

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$$

is a **polynomial function of degree** $\boldsymbol{n}$.

The **leading coefficient** is a_n.

The zero function $f(x) = 0$ is a polynomial function.

It has no degree and no leading coefficient.

Polynomial Functions of No and Low Degree

Name	Form	Degree
Zero Function	$f(x) = 0$	Undefined
Constant Function	$f(x) = a \ \ (a \neq 0)$	0
Linear Function	$f(x) = ax + b \ (a \neq 0)$	1
Quadratic Function	$f(x) = ax^2 + bx + c \ (a \neq 0)$	2

Example Finding an Equation of a Linear Function

Write an equation for the linear function f such that $f(-1)=2$ and $f(2)=3$.

Example **Finding an Equation of a Linear Function**

Write an equation for the linear function f such that $f(-1) = 2$ and $f(2) = 3$.

The line contains the points $(-1, 2)$ and $(2, 3)$. Find the slope:

$$m = \frac{3-2}{2+1} = \frac{1}{3}$$

Use the point-slope formula and the point $(2, 3)$:

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{1}{3}(x - 2)$$

$$y - 3 = \frac{1}{3}x - \frac{2}{3}$$

$$y = \frac{1}{3}x + \frac{7}{3}$$

$$f(x) = \frac{1}{3}x + \frac{7}{3}$$

Average Rate of Change

The average rate of change of a function $y = f(x)$ between $x = a$ and $x = b$, $a \neq b$, is

$$\frac{f(b) - f(a)}{b - a}.$$

Constant Rate of Change Theorem

A function defined on all real numbers is a linear function if and only if it has a constant nonzero average rate of change between any two points on its graph.

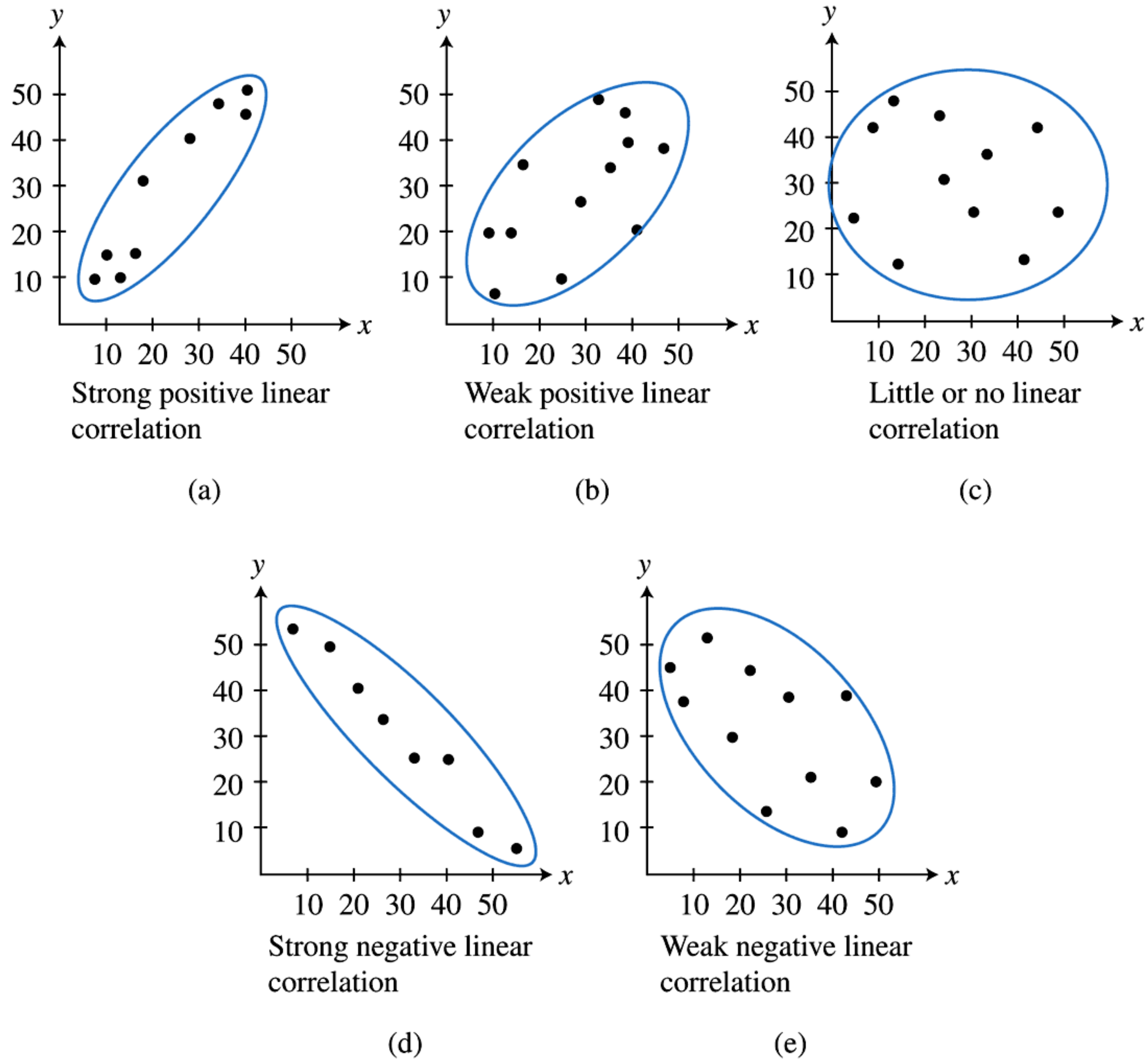
Characterizing the Nature of a Linear Function

Point of View	**Characterization**
Verbal	polynomial of degree 1
Algebraic	$f(x) = mx + b \ (m \neq 0)$
Graphical	slant line with slope m, y-intercept b
Analytical	function with constant nonzero rate of change m: f is increasing if $m > 0$, decreasing if $m < 0$; initial value of the function = $f(0) = b$

Properties of the Correlation Coefficient, r

1. $-1 \leq r \leq 1$
2. When $r > 0$, there is a positive linear correlation.
3. When $r < 0$, there is a negative linear correlation.
4. When $|r| \approx 1$, there is a strong linear correlation.
5. When $|r| \approx 0$, there is weak or no linear correlation.

Linear Correlation



Regression Analysis

1. Enter and plot the data (scatter plot).
2. Find the regression model that fits the problem situation.
3. Superimpose the graph of the regression model on the scatter plot, and observe the fit.
4. Use the regression model to make the predictions called for in the problem.

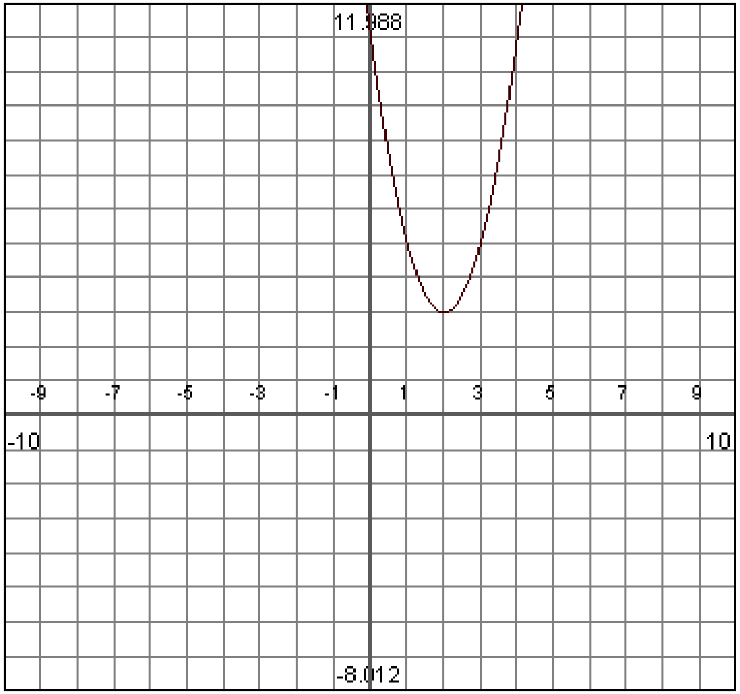
Example **Transforming the Squaring Function**

Describe how to transform the graph of $f(x)=x^2$ into the graph of $f(x)=2(x-2)^2+3$.

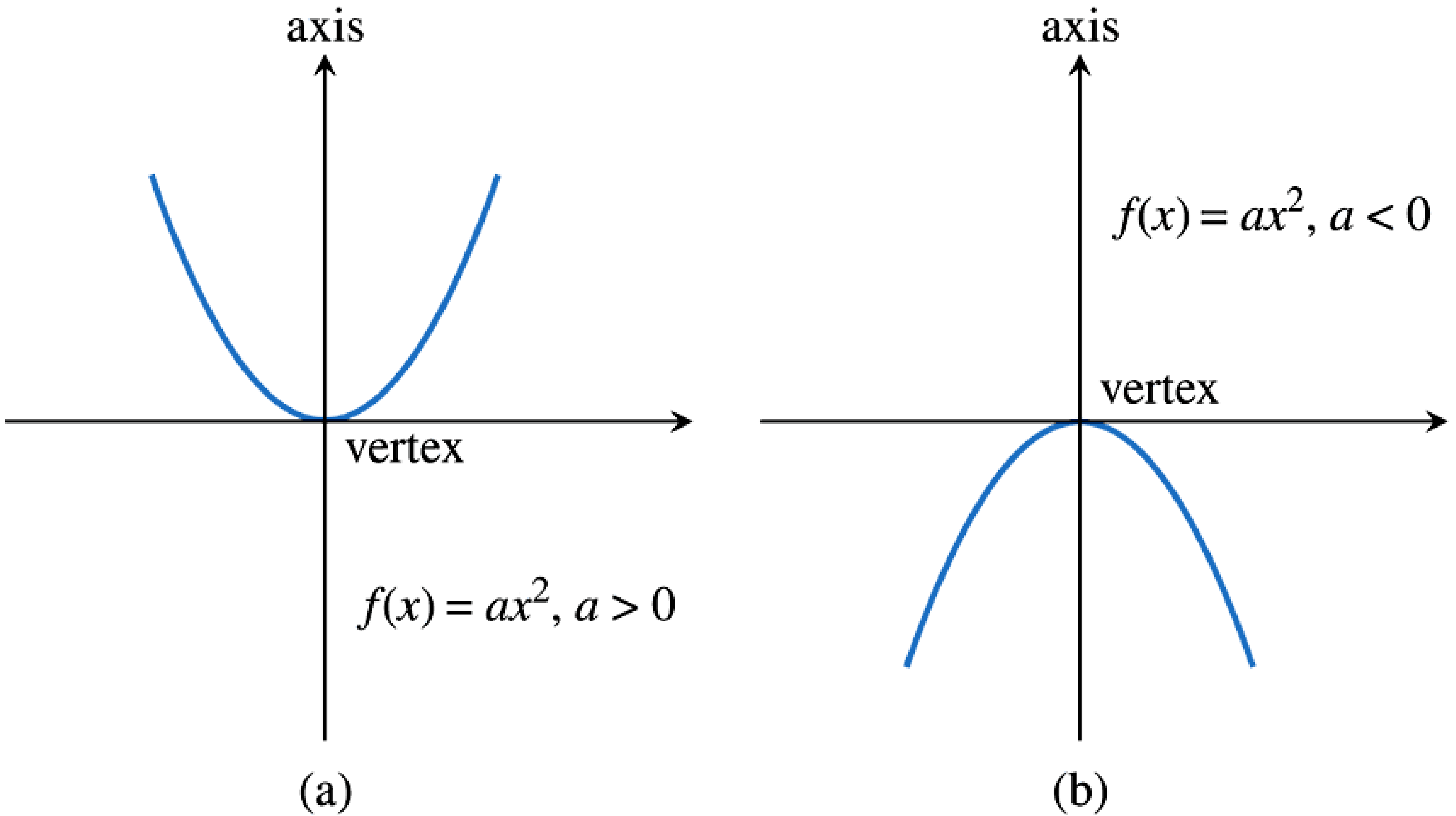
Example Transforming the Squaring Function

Describe how to transform the graph of $f(x) = x^2$ into the graph of $f(x) = 2(x-2)^2 + 3$.

The graph of $f(x) = 2(x-2)^2 + 3$ is obtained by vertically stretching the graph of $f(x) = x^2$ by a factor of 2 and translating the resulting graph 2 units right and 3 units up.



The Graph of $f(x)=ax^2$



Vertex Form of a Quadratic Equation

Any quadratic function $f(x) = ax^2 + bx + c$, $a \neq 0$, can be written in the **vertex form**

$$f(x) = a(x - h)^2 + k$$

The graph of f is a parabola with vertex (h, k) and axis $x = h$, where $h = -b/(2a)$ and $k = c - ah^2$. If $a > 0$, the parabola opens upward, and if $a < 0$, it opens downward.

Example **Finding the Vertex and Axis of a Quadratic Function**

Use the vertex form of a quadratic function to find the vertex and axis of the graph of $f(x) = 2x^2 - 8x + 11$. Rewrite the equation in vertex form.

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The standard polynomial form of f is $f(x) = 2x^2 - 8x + 11$; $a = 2,\ b = -8,\ c = 11$, and the coordinates of the vertex are

$$h = -\frac{b}{2a} = \frac{8}{4} = 2 \text{ and } k = f(h) = f(2) = 2(2)^2 - 8(2) + 11 = 3.$$

The equation of the axis is $x = 2$, the vertex is (2,3), and the vertex form of f is $f(x) = 2(x-2)^2 + 3$.

Example Using Algebra to Describe the Graph of a Quadratic Function

Use completing the square to describe the graph of

$f(x) = -4x^2 + 12x - 8.$

Support your answer graphically.

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$$\begin{aligned} f(x) &= -4x^2 + 12x - 8 \\ &= -4\left(x^2 - 3x\right) - 8 \\ &= -4\left(x^2 - 3x + (\ \ ) - (\ \ )\right) - 8 \\ &= -4\left(x^2 - 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right) - 8 \end{aligned}$$

Example Using Algebra to Describe the Graph of a Quadratic Function

Use completing the square to describe the graph of

$f(x) = -4x^2 + 12x - 8.$

Support your answer graphically.

$$= -4\left(x^2 - 3x + \frac{9}{4}\right) - (-4)\left(\frac{9}{4}\right) - 8$$

$$= -4\left(x - \frac{3}{2}\right)^2 + 1$$

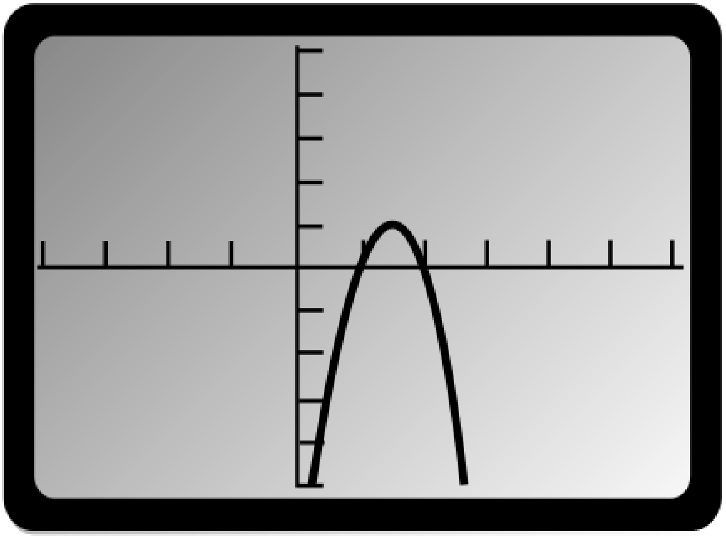
Example Using Algebra to Describe the Graph of a Quadratic Function

Use completing the square to describe the graph of

$$f(x) = -4x^2 + 12x - 8.$$

Support your answer graphically.

The graph of *f* is a downward-opening parabola with vertex (3/2, 1) and axis of symmetry *x* = 3/2. The *x*-intercepts are at *x* = 1 and *x* = 2.



$[-4, 6]$ by $[-5, 5]$

Characterizing the Nature of a Quadratic Function

Point of View	**Characterization**
Verbal	polynomial of degree 2
Algebraic	$f(x) = ax^2 + bx + c$ or $f(x) = a(x - h)^2 + k \ (a \neq 0)$
Graphical	parabola with vertex (h, k) and axis $x = h$; opens upward if $a > 0$, opens downward if $a < 0$; initial value = y-intercept = $f(0) = c$; $x\text{-intercepts} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vertical Free-Fall Motion

The **height** s and vertical **velocity** v of an object in free fall are given by

$$s(t) = -\frac{1}{2}gt^2 + v_0 t + s_0 \quad \text{and} \quad v(t) = -gt + v_0,$$

where t is time (in seconds), $g \approx 32 \text{ ft/sec}^2 \approx 9.8 \text{ m/sec}^2$ is the **acceleration due to gravity**, v_0 is the *initial vertical velocity* of the object, and s_0 is its *initial height*.

Quick Review

1. Write an equation in slope-intercept form for a line with slope $m = 2$ and y-intercept 10.
2. Write an equation for the line containing the points $(-2, 3)$ and $(3,4)$.
3. Expand $(x+6)^2$.
4. Expand $(2x-3)^2$.
5. Factor $2x^2+8x+8$.

Quick Review Solutions

1. Write an equation in slope-intercept form for a line with slope $m = 2$ and y-intercept 10. $y = 2x + 10$
2. Write an equation for the line containing the points $(-2, 3)$ and $(3,4)$. $y - 4 = \frac{1}{5}(x - 3)$
3. Expand $(x + 6)^2$. $x^2 + 12x + 36$
4. Expand $(2x - 3)^2$. $4x^2 - 12x + 9$
5. Factor $2x^2 + 8x + 8$. $2(x + 2)^2$