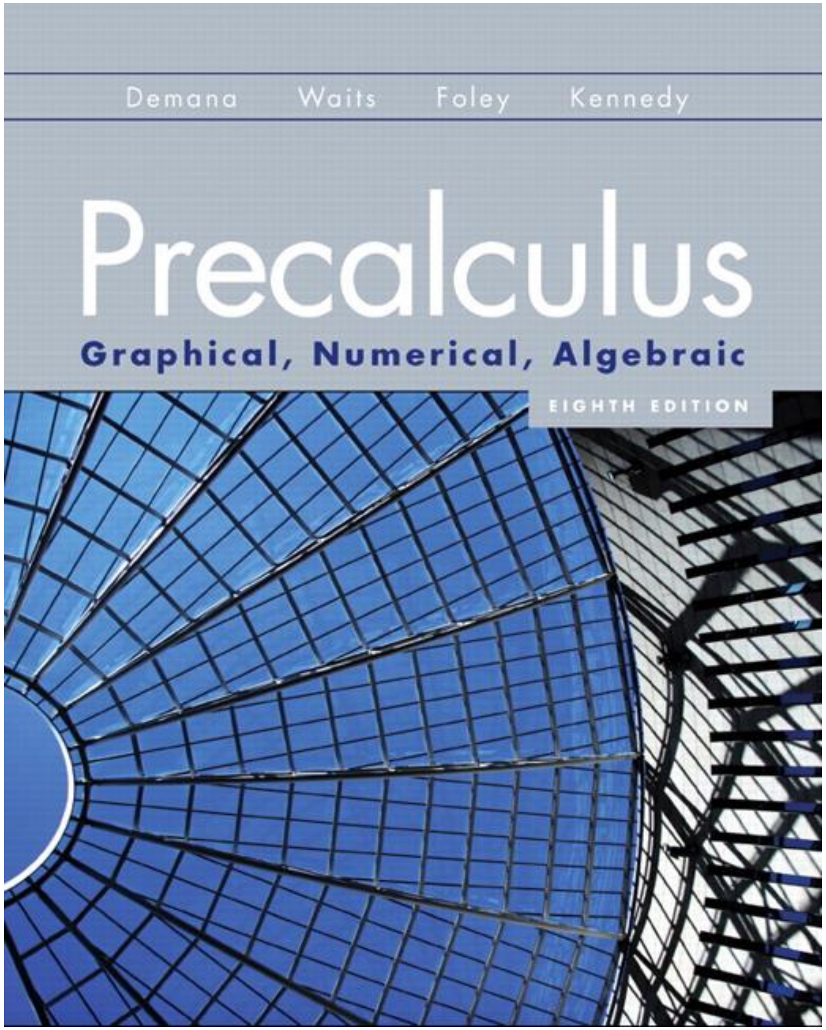
1.3

Twelve Basic Functions



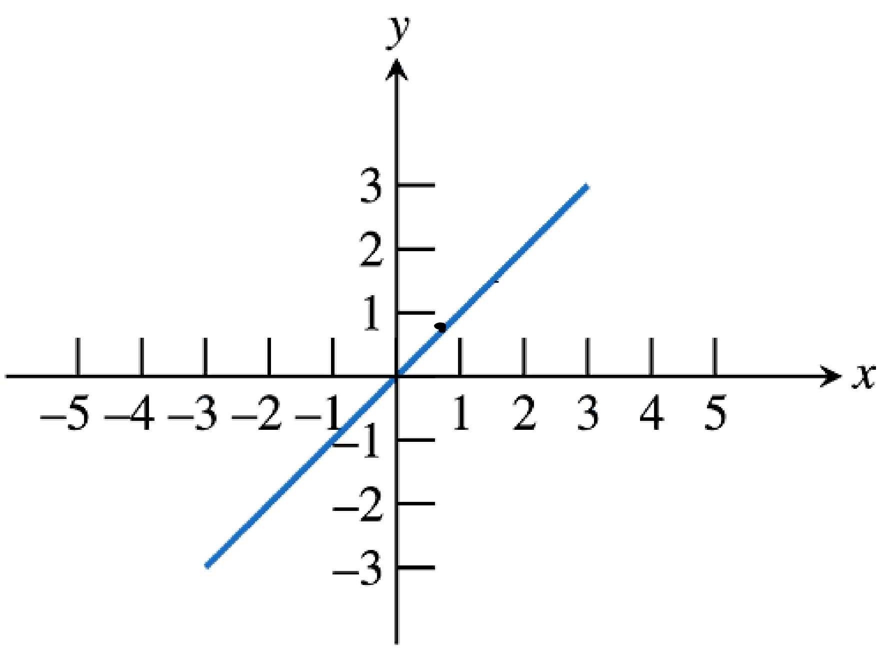
What you'll learn about

- What Graphs can Tell Us
- Twelve Basic Functions
- Analyzing Functions Graphically

… and why

As you continue to study mathematics, you will find that the twelve basic functions presented here will come up again and again. By knowing their basic properties, you will recognize them when you see them.

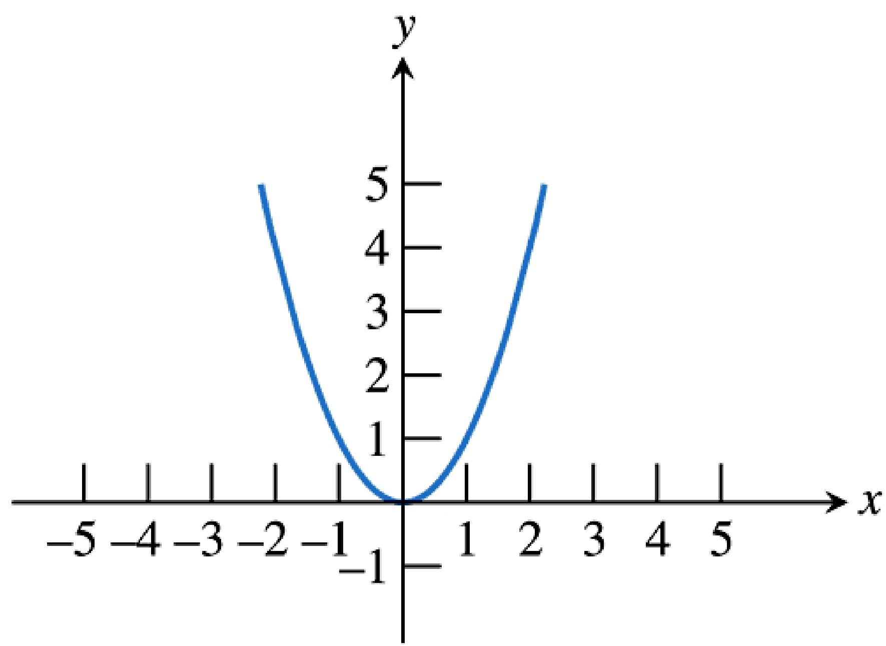
The Identity Function



$f(x) = x$

Interesting fact: This is the only function that acts on every real number by leaving it alone.

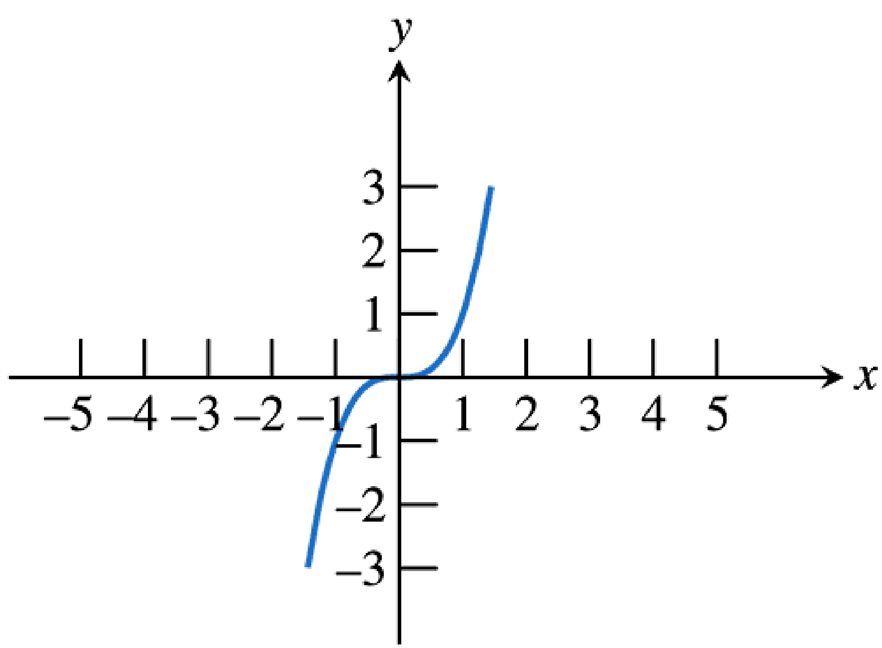
The Squaring Function



$$f(x) = x^2$$

Interesting fact: The graph of this function, called a parabola, has a reflection property that is useful in making flashlights and satellite dishes.

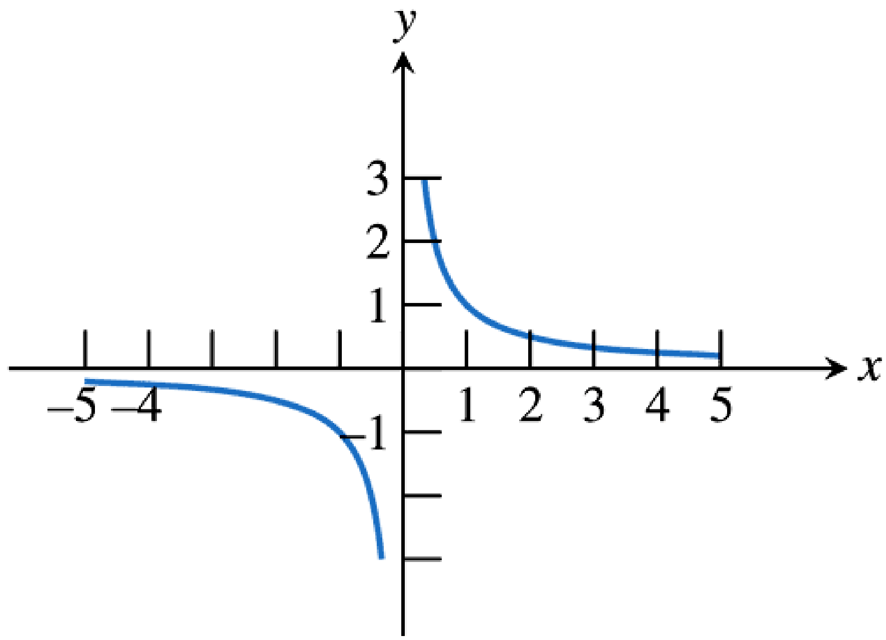
The Cubing Function



$$f(x) = x^3$$

Interesting fact: The origin is called a "point of inflection" for this curve because the graph changes curvature at that point.

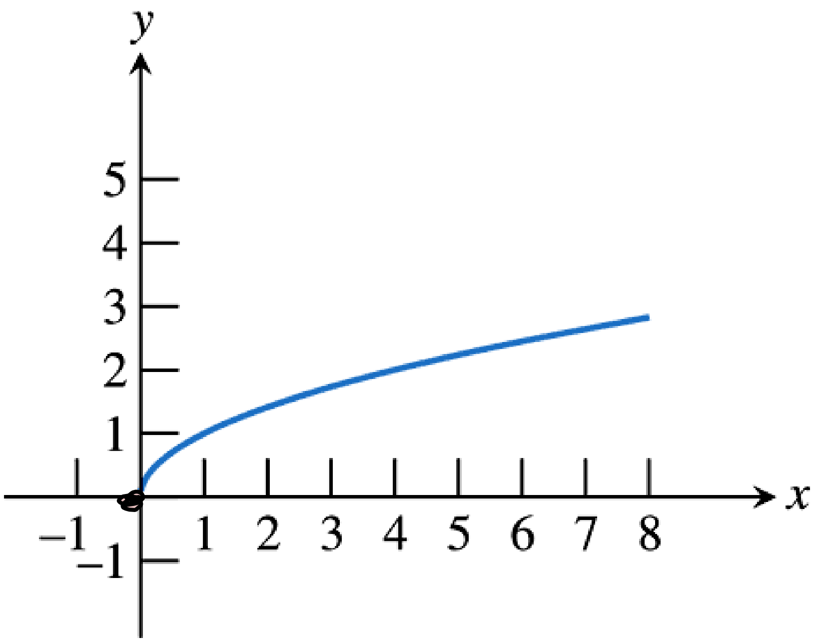
The Reciprocal Function



$$f(x) = \frac{1}{x}$$

Interesting fact: This curve, called a hyperbola, also has a reflection property that is useful in satellite dishes.

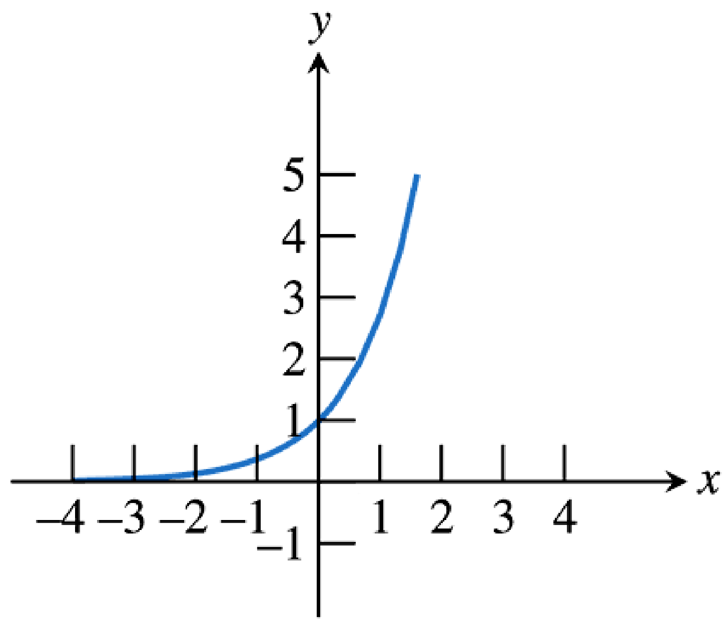
The Square Root Function



$$f(x) = \sqrt{x}$$

Interesting fact: Put any positive number into your calculator. Take the square root. Then take the square root again. Then take the square root again, and so on. Eventually you will always get 1.

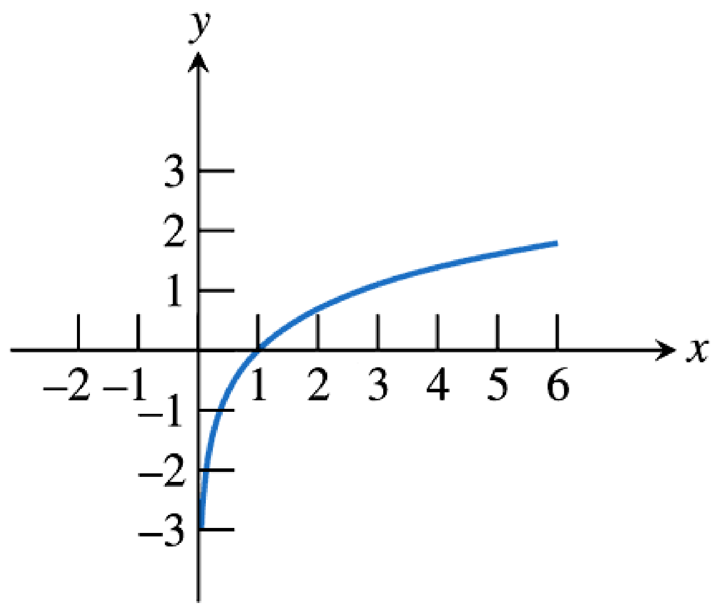
The Exponential Function



$$f(x) = e^x$$

Interesting fact: The number e is an irrational number (like π) that shows up in a variety of applications. The symbols e and π were both brought into popular use by the great Swiss mathematician Leonhard Euler (1707–1783).

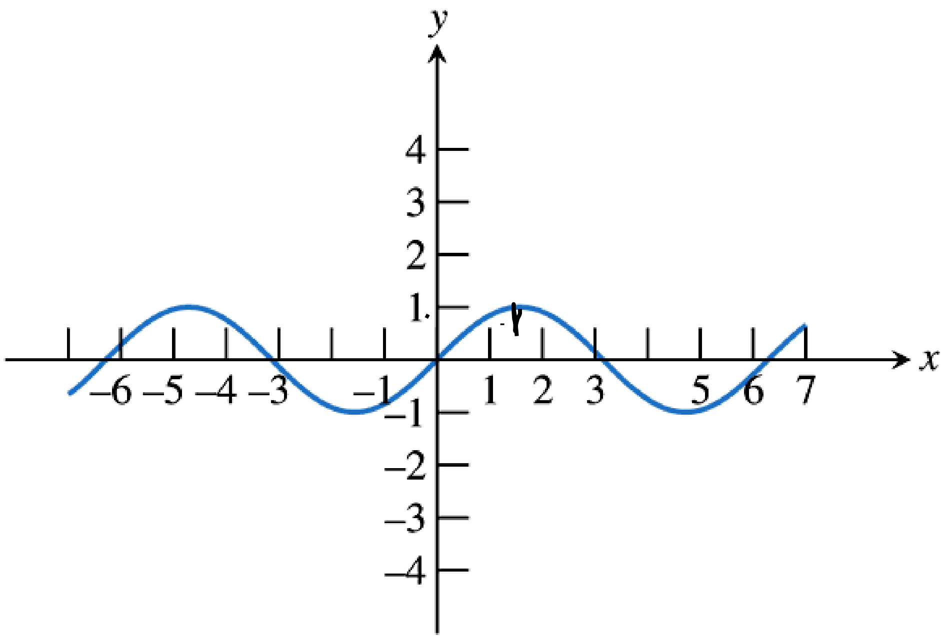
The Natural Logarithm Function



$$f(x) = \ln x$$

Interesting fact: This function increases very slowly. If the *x*-axis and *y*-axis were both scaled with unit lengths of one inch, you would have to travel more than two and a half miles along the curve just to get a foot above the *x*-axis.

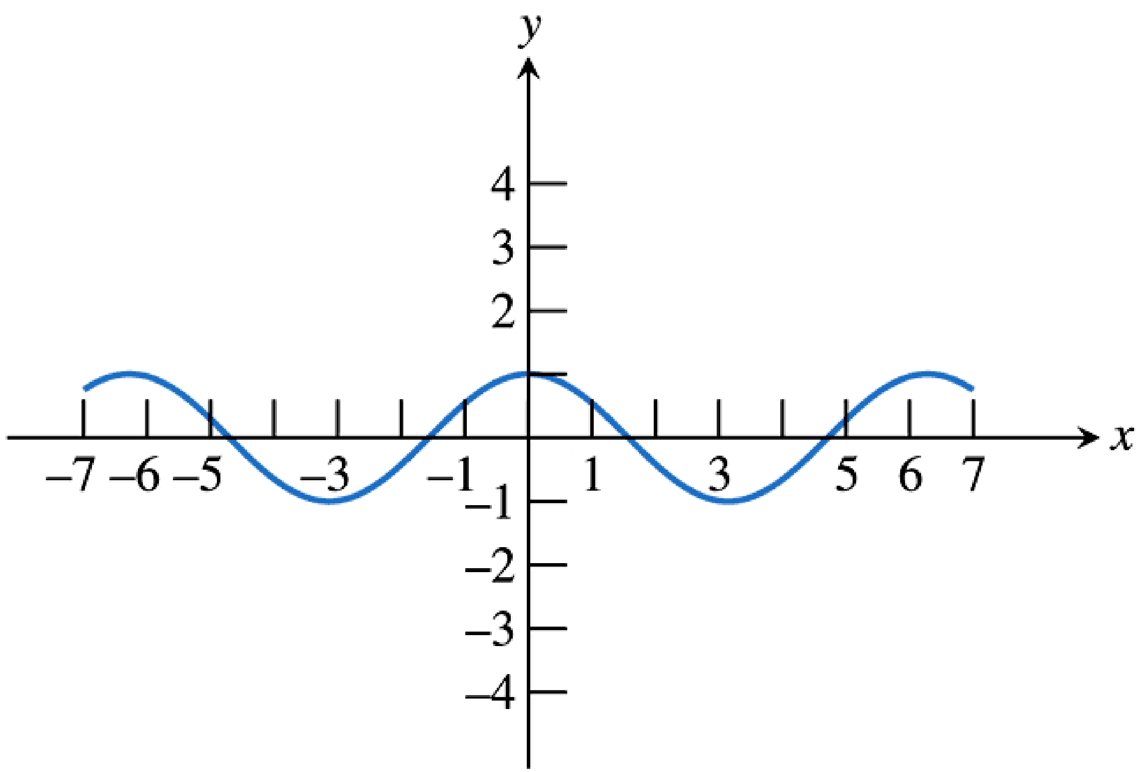
The Sine Function



$f(x) = \sin x$

Interesting fact: This function and the sinus cavities in your head derive their names from a common root: the Latin word for "bay." This is due to a 12th-century mistake made by Robert of Chester, who translated a word incorrectly from an Arabic manuscript.

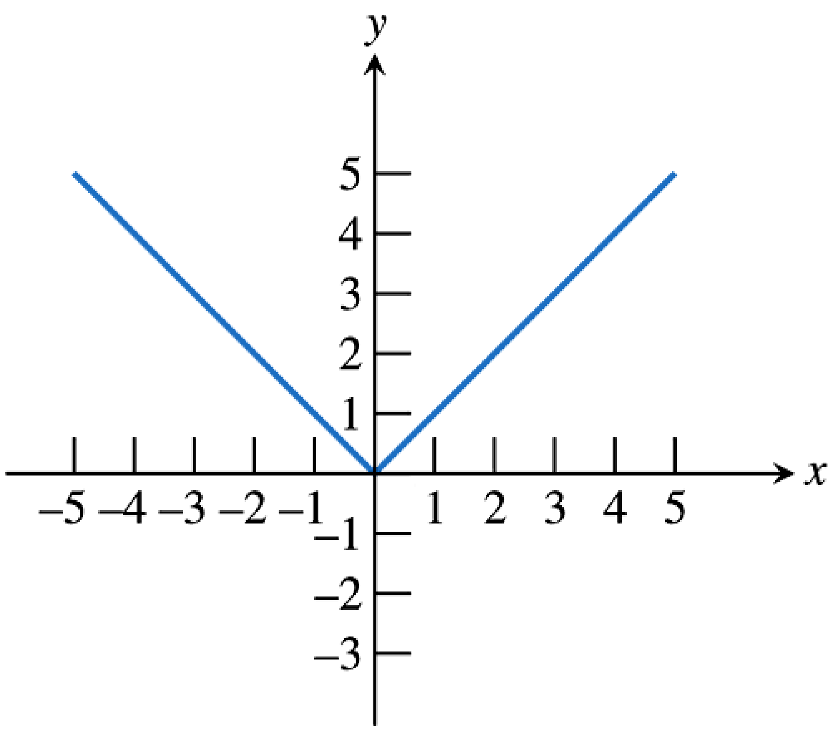
The Cosine Function



$f(x) = \cos x$

Interesting fact: The local extrema of the cosine function occur exactly at the zeros of the sine function, and vice versa.

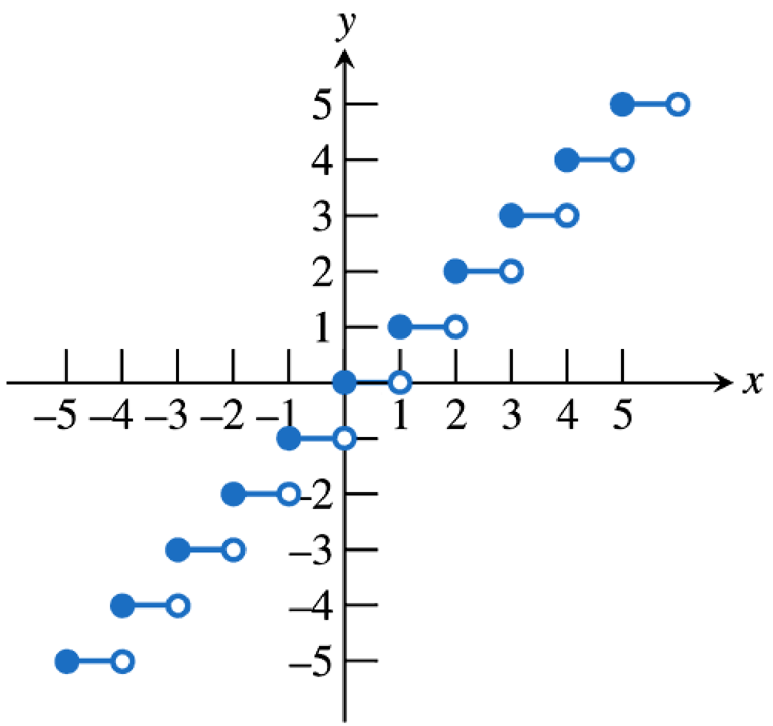
The Absolute Value Function



$$f(x) = |x| = \text{abs}(x)$$

Interesting fact: This function has an abrupt change of direction (a “corner”) at the origin, while our other functions are all “smooth” on their domains.

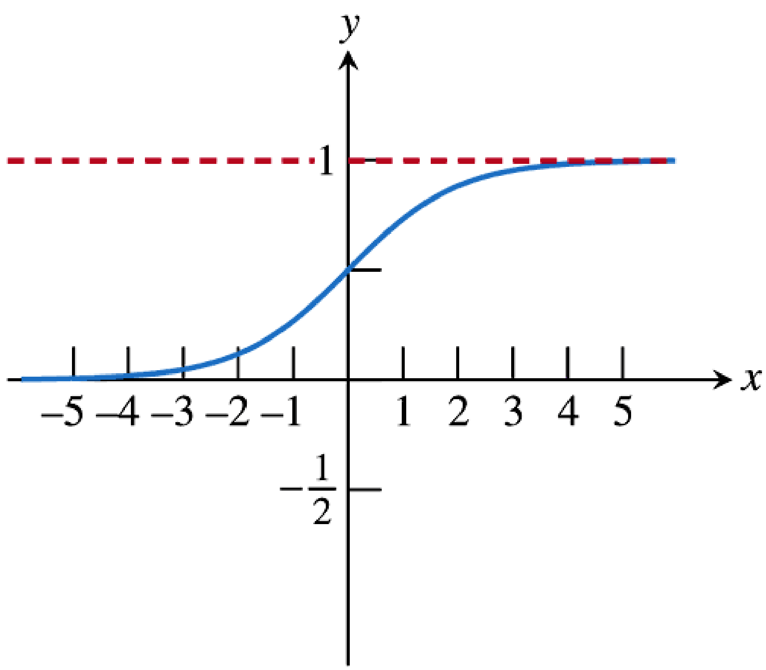
The Greatest Integer Function



$f(x) = \text{int}(x)$

Interesting fact: This function has a jump discontinuity at every integer value of x. Similar-looking functions are called *step functions.*

The Logistic Function



$$f(x) = \frac{1}{1 + e^{-x}}$$

Interesting fact: There are two horizontal asymptotes, the x-axis and the line $y = 1$. This function provides a model for many applications in biology and business.

Example **Looking for Domains**

One of the functions has domain the set of all reals except 0.

Which function is it?

Solution

One of the functions has domain the set of all reals except 0.

Which function is it?

The function $y = 1/x$ has a vertical asymptote at $x = 0$.

Example **Analyzing a Function Graphically**

Graph the function $y = -x^2 - 1$.

Then answer the following questions.

(a) On what interval is the function increasing? decreasing?

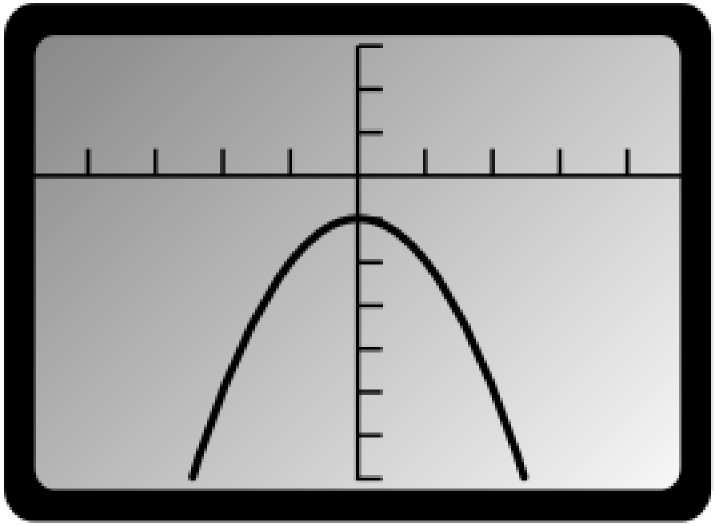
(b) Is the function even, odd, or neither?

(c) Does the function have any extrema?

(d) How does the graph relate to the graph of the basic function $y = x^2$?

Solution

Graph the function $y = -x^2 - 1$.



$[-4.7, 4.7]$ by $[-7, 3]$

(a) Increasing on $(-\infty, 0]$ and decreasing $[0, \infty)$

(b) Symmetric with respect to the y-axis, but not the x-axis so the function even.

(c) Maximum value of -1 at $x = 0$

(d) The graph is the grpah of $y = x^2$ reflected over the x-axis and translated 1 unit down.

Quick Review

Evaluate the expressions without using a calculator.

1. $\left| -43.21 \right|$
2. $\left| \pi - 4 \right|$
3. $\ln 1$
4. e^0
5. $\left(\sqrt[4]{4} \right)^4$

Quick Review Solutions

Evaluate the expressions without using a calculator.

1. $\left|-43.21\right| = 43.21$
2. $\left|\pi - 4\right| = 4 - \pi$
3. $\ln\ 1 = 0$
4. $e^0 = 1$
5. $\left(\sqrt[4]{4}\right)^4 = 4$