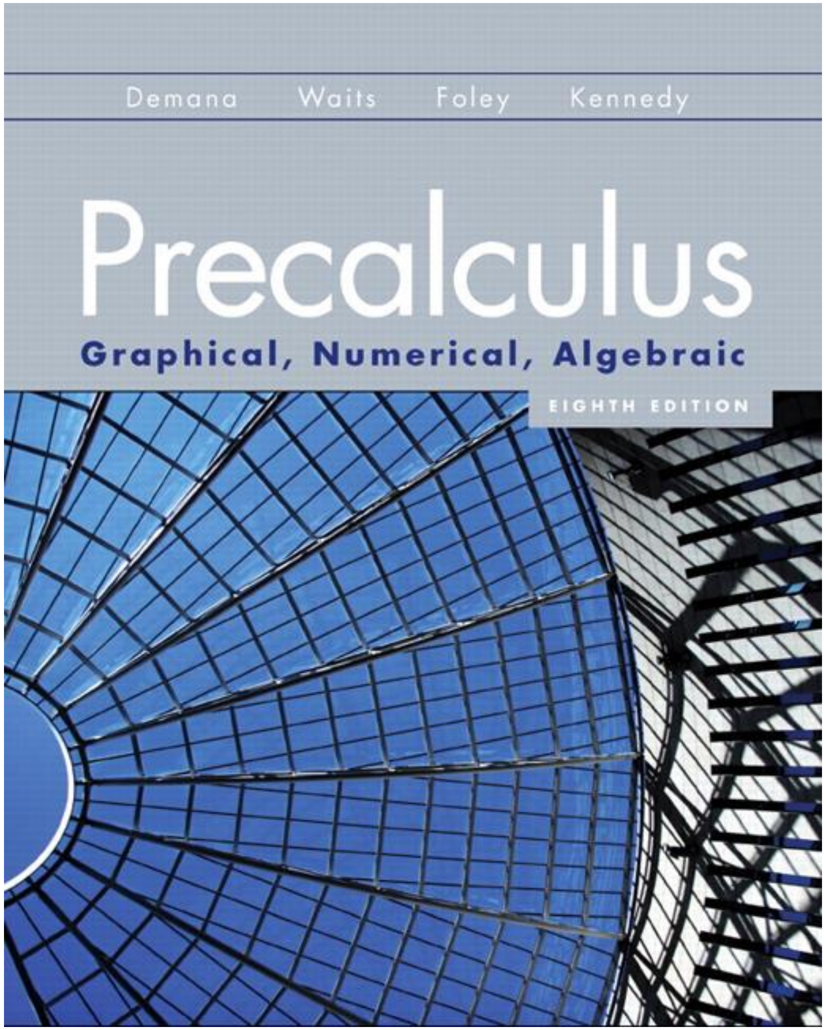
1.1

Modeling and Equation Solving



What you’ll learn about

- Numeric Models
- Algebraic Models
- Graphic Models
- The Zero Factor Property
- Problem Solving
- Grapher Failure and Hidden Behavior
- A Word About Proof

… and why

Numerical, algebraic, and graphical models provide different methods to visualize, analyze, and understand data.

Mathematical Model

A **mathematical model** is a mathematical structure that approximates phenomena for the purpose of studying or predicting their behavior.

Numeric Model

A **numeric model** is a kind of mathematical model in which numbers (or data) are analyzed to gain insights into phenomena.

Algebraic Model

An **algebraic model** uses formulas to relate variable quantities associated with the phenomena being studied.

Example Comparing Pizzas

A pizzeria sells a rectangular 20" by 22" pizza for the same price as itslarge round pizza (24" diameter). If both pizzas are the same thickness, which option gives the most pizza for the money?

Solution

A pizzeria sells a rectangular 20" by 22" pizza for the same price as its large round pizza (24" diameter). If both pizzas are the same thickness, which option gives the most pizza for the money?

Compare the areas of the pizzas.

Rectangular pizza: Area $= l \times w = 20 \times 22 = 440$ square inches

Circular pizza: Area $= \pi r^2 = \pi \left(\frac{24}{2} \right)^2 = 144\pi \approx 452.4$ square inches

The round pizza is larger and therefore gives more for the money.

Graphical Model

A **graphical model** is a visible representation of a numerical model or an algebraic model that gives insight into the relationships between variable quantities.

The Zero Factor Property

A product of real numbers is zero if and only if at least one of the factors in the product is zero.

Example **Solving an Equation**

Solve the equation $x^2 = 8 - 4x$ algebraically.

Solution

Solve the equation $x^2 = 8 - 4x$ algebraically.

Set the given equation equal to zero:

$$x^2 + 4x - 8 = 0$$

Use the quadratic formula to solve for x.

$$x = \frac{-4 \pm \sqrt{16 + 32}}{2} = -2 \pm 2\sqrt{3}$$

$$x = -2 - 2\sqrt{3} \approx -5.464101615$$

and

$$x = -2 + 2\sqrt{3} \approx 1.464101615$$

Fundamental Connection

If a is a real number that solves the equation

$f(x) = 0$,

then these three statements are equivalent:

1. The number a is a **root** (or **solution**) of the **equation** $f(x) = 0$.
2. The number a is a **zero** of $y = f(x)$.
3. The number a is an ***x*-intercept** of the **graph** of $y = f(x)$. (Sometimes the point $(a, 0)$ is referred to as an x-intercept.)

Pólya's Four Problem-Solving Steps

1. Understand the problem.
2. Devise a plan.
3. Carry out the plan.
4. Look back.

A Problem-Solving Process

Step 1 – Understand the problem.

- Read the problem as stated, several times if necessary.
- Be sure you understand the meaning of each term used.
- Restate the problem in your own words. Discuss the problem with others if you can.
- Identify clearly the information that you need to solve the problem.
- Find the information you need from the given data.

A Problem-Solving Process

Step 2 – Develop a mathematical model of the problem.

- Draw a picture to visualize the problem situation. It usually helps.
- Introduce a variable to represent the quantity you seek. (There may be more than one.)
- Use the statement of the problem to find an equation or inequality that relates the variables you seek to quantities that you know.

A Problem-Solving Process

Step 3 – Solve the mathematical model and support or confirm the solution.

- Solve algebraically using traditional algebraic models and support graphically or support numerically using a graphing utility.
- Solve graphically or numerically using a graphing utility and confirm algebraically using traditional algebraic methods.
- Solve graphically or numerically because there is no other way possible.

A Problem-Solving Process

Step 4 – Interpret the solution in the problem setting.

- Translate your mathematical result into the problem setting and decide whether the result makes sense.

Example **Seeing Grapher Failure**

□□□□□□□□□□□□□□□□□ of $y = 3/(2x - 5)$ on a graphing calculator. Is there an x-intercept?

Solution

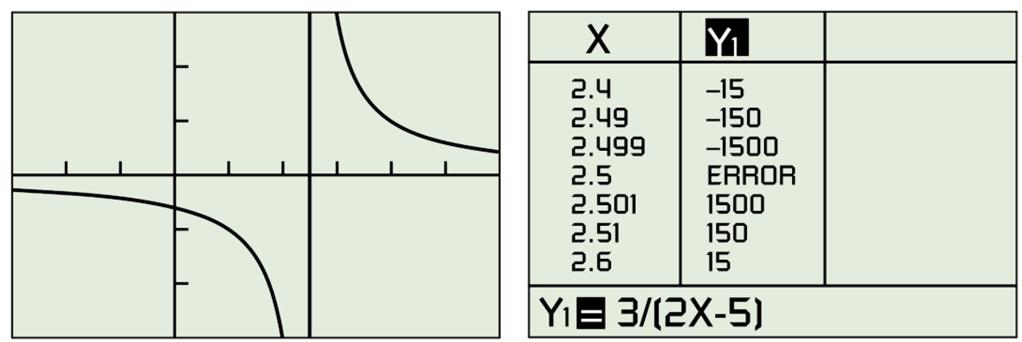
□□□□□□□□□□□□□□□□□ of $y = 3/(2x - 5)$ on a graphing calculator. Is there an x-intercept?

Notice that the graph appears to show an x-intercept between 2 and 3. Confirm this algebraically.

$$0 = \frac{3}{2x-5}$$

$$0(2x-5) = 3$$

$$0 = 3$$



[–3, 6] by [–3, 3]

(a) (b)

This statement is false for all x, so there is no x-intercept. The grapher shows a vertical asymptote at $x = 2.5$.

Quick Review

Factor the following expressions completely over the real numbers.

1. $x^2 - 9$
2. $4y^2 - 81$
3. $x^2 - 8x + 16$
4. $2x^2 + 7x + 3$
5. $x^4 - 3x^2 - 4$

Quick Review Solutions

Factor the following expressions completely over the real numbers.

1. $x^2 - 9$ $(x+3)(x-3)$
2. $4y^2 - 81$ $(2y+9)(2y-9)$
3. $x^2 - 8x + 16$ $(x-4)^2$
4. $2x^2 + 7x + 3$ $(2x+1)(x+3)$
5. $x^4 - 3x^2 - 4$ $(x^2+1)(x-2)(x+2)$