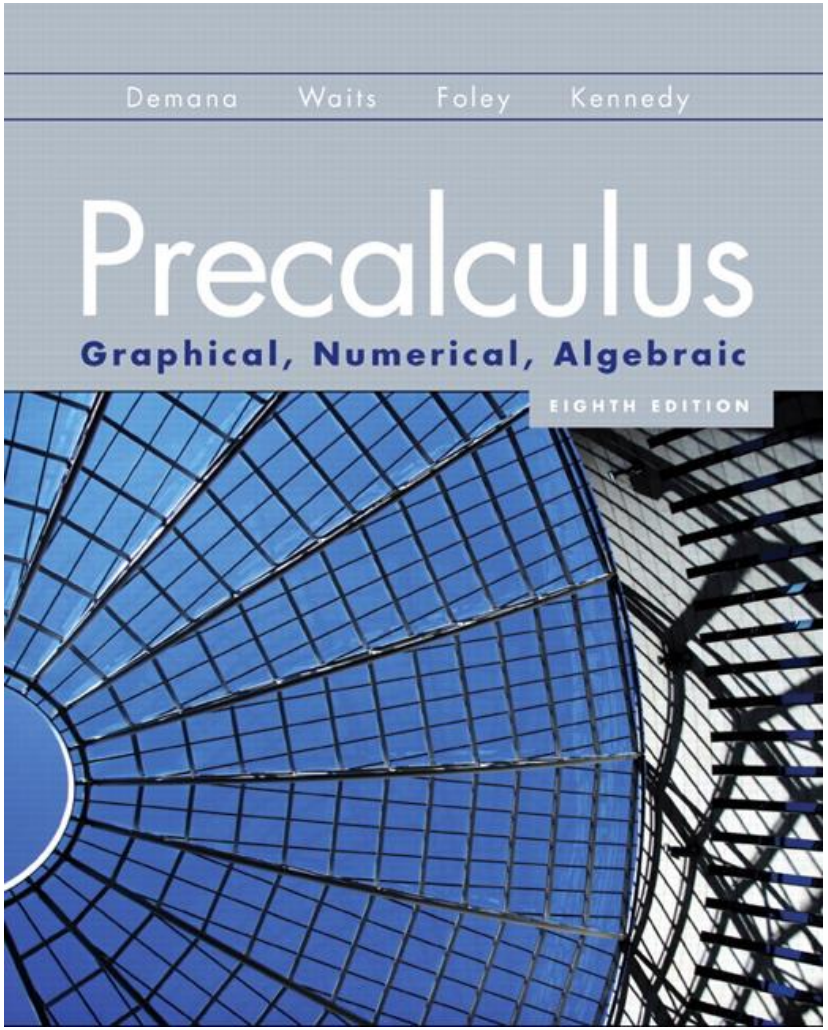
P.7

Solving Inequalities Algebraically and Graphically



What you'll learn about

- Solving Absolute Value Inequalities
- Solving Quadratic Inequalities
- Approximating Solutions to Inequalities
- Projectile Motion

… and why

These techniques are involved in using a graphing utility to solve inequalities in this textbook.

Solving Absolute Value Inequalities

Let u be an algebraic expression in x and let a be a real number with $a \geq 0$.

1. If $|u| < a$, then u is in the interval $(-a, a)$. That is,

$$|u| < a \quad \text{if and only if} \quad -a < u < a.$$

2. If $|u| > a$, then u is in the interval $(-\infty, -a)$ or (a, ∞). That is,

$$|u| > a \quad \text{if and only if} \quad u < -a \text{ or } u > a.$$

The inequalities $<$ and $>$ can be replaced with $\leq$ and $\geq$, respectively.

Example **Solving an Absolute Value Inequality**

Solve $|x+3|<5$.

Solution

Solve $|x+3|<5$.

$$|x+3|<5$$

$$-5<x+3<5$$

$$-8<x<2$$

As an interval the solution in $(-8,2)$.

Example **Solving a Quadratic Inequality**

Solve $5x^2 - 11x \geq 12.$

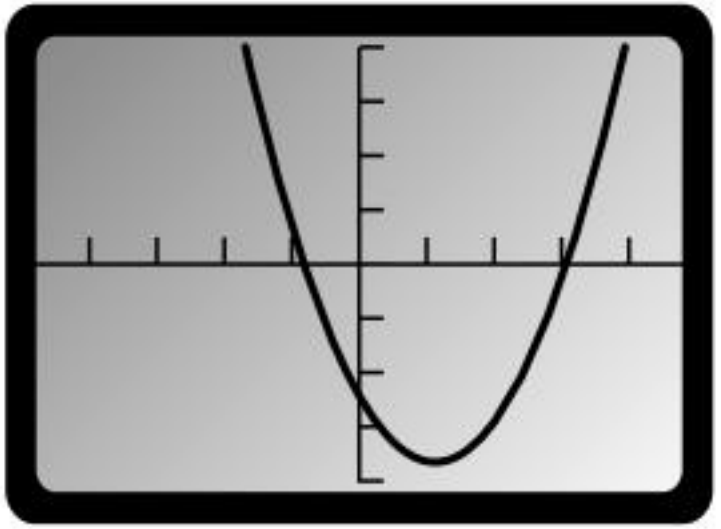
Solution

$$5x^2 - 11x - 12 \geq 0$$

$$5x^2 - 11x - 12 = 0$$

$$(5x + 4)(x - 3) = 0$$

$$x = -\frac{4}{5} \text{ or } x = 3$$



Use these solutions and a sketch of the equation $y = 5x^2 - 11x - 12$ to find the solution to the inequality in interval form $\left(-\infty, -\frac{4}{5}\right] \cup \left[3, \infty\right)$.

Projectile Motion

Suppose an object is launched vertically from a point s_0 feet above the ground with an initial velocity of v_0 feet per second. The vertical position s (in feet) of the object t seconds after it is launched is

$$s = -16t^2 + v_0 t + s_0.$$

Example **Finding Height of a Projectile**

A projectile is launched straight up from ground level with an initial velocity of 288ft/sec.

(a) When will the projectile's height above ground be 1152 ft?

(b) When will the projectile's height above ground be at least 1152 ft?

Solution

Here $s_0 = 0$ and $v_0 = 288$. So the projectile's height is $S = -16t^2 + 288t$.

(a) Determine when $s = 1152$.

$$s = -16t^2 + 288t$$

$$1152 = -16t^2 + 288t$$

$$16t^2 - 288t + 1152 = 0$$

$$t^2 - 18t + 72 = 0$$

$$(t-6)(t-12) = 0$$

$$t = 6 \quad \text{or} \quad t = 12$$

Determine when $s = 1152$.

Solution continued

The projectile is 1152 ft above ground twice; the first time at $t = 6$ sec on the way up, and the second time at $t = 12$ sec on the way down.

Solution continued

(b) The projectile will be at least 1152 ft above ground when $s \geq 1152$. We can see from the figure together with the algebraic work in (a) that the solution is [6, 12]. This means that the projectile is at least 1152 ft above ground for times between $t = 6$ sec and $t = 12$ sec, including 6 and 12 sec.

Quick Review

Solve for x.

1. $-3 < 2x + 1 < 9$
2. $|2x + 1| = 3$
3. Factor completely. $4x^2 - 9$
4. Reduce the fraction to lowest terms. $\dfrac{x^2 - 49}{x^2 + 7x}$
5. Add the fractions and simplify. $\dfrac{x}{x+1} + \dfrac{x+2}{x}$

Quick Review Solutions

Solve for x.

1. $-3 < 2x+1 < 9$ $\quad -2 < x < 4$

2. $|2x+1| = 3$ $\quad x = -2$ or $x = 1$

3. Factor completely. $4x^2 - 9$ $\quad (2x-3)(2x+3)$

4. Reduce the fraction to lowest terms. $\dfrac{x^2-49}{x^2+7x}$ $\quad \dfrac{x-7}{x}$

5. Add the fractions and simplify. $\dfrac{x}{x+1} + \dfrac{x+2}{x}$ $\quad \dfrac{2x^2+3x+2}{x^2+x}$

Chapter Test

1. Write the number in scientific notation.

The diameter of a red blood corpuscle is about 0.000007 meter.

2. Find the standard form equation for the circle with center $(5, -3)$ and radius 4.

3. Find the slope of the line through the points $(-1, -2)$ and $(4, -5)$.

4. Find the equation of the line through $(2, -3)$ and perpendicular to the line $2x + 5y = 3$.

5. Solve the equation algebraically. $\frac{x-2}{3} + \frac{x+5}{2} = \frac{1}{3}$

6. Solve the equation algebraically. $6x^2 + 7x = 3$

Chapter Test

7. Solve the equation algebraically. $|4x+1|=3$

8. Solve the inequality. $|3x+4|\geq 2$

9. Solve the inequality. $4x^2+12x+9\geq 0$

10. Perform the indicated operation, and write the result in standard form. $(5-7i)-(3-2i)$

Chapter Test Solutions

1. Write the number in scientific notation.

The diameter of a red blood corpuscle is about 0.000007 meter. 7×10^{-6}

2. Find the standard form equation for the circle with center $(5,-3)$ and radius 4. $(x-5)^2+(y+3)^2=16$

3. Find the slope of the line through the points $(-1,-2)$ and $(4,-5)$. $-\frac{3}{5}$

4. Find the equation of the line through $(2,-3)$ and perpendicular to the line $2x+5y=3$. $y=\frac{5}{2}x-8$

5. Solve the equation algebraically. $\frac{x-2}{3}+\frac{x+5}{2}=\frac{1}{3}$ $x=-\frac{9}{5}$

6. Solve the equation algebraically. $6x^2+7x=3$ $x=\frac{1}{3}$ or $x=-\frac{3}{2}$

Chapter Test Solutions

7. Solve the equation algebraically. $|4x+1|=3$ $x=\frac{1}{2}$ or $x=-1$

8. Solve the inequality. $|3x+4|\geq 2$ $(-\infty,-2]\cup\left[-\frac{2}{3},\infty\right)$

9. Solve the inequality. $4x^2+12x+9\geq 0$ $(-\infty,\infty)$

10. Perform the indicated operation, and write the result in standard form. $(5-7i)-(3-2i)$ $2-5i$