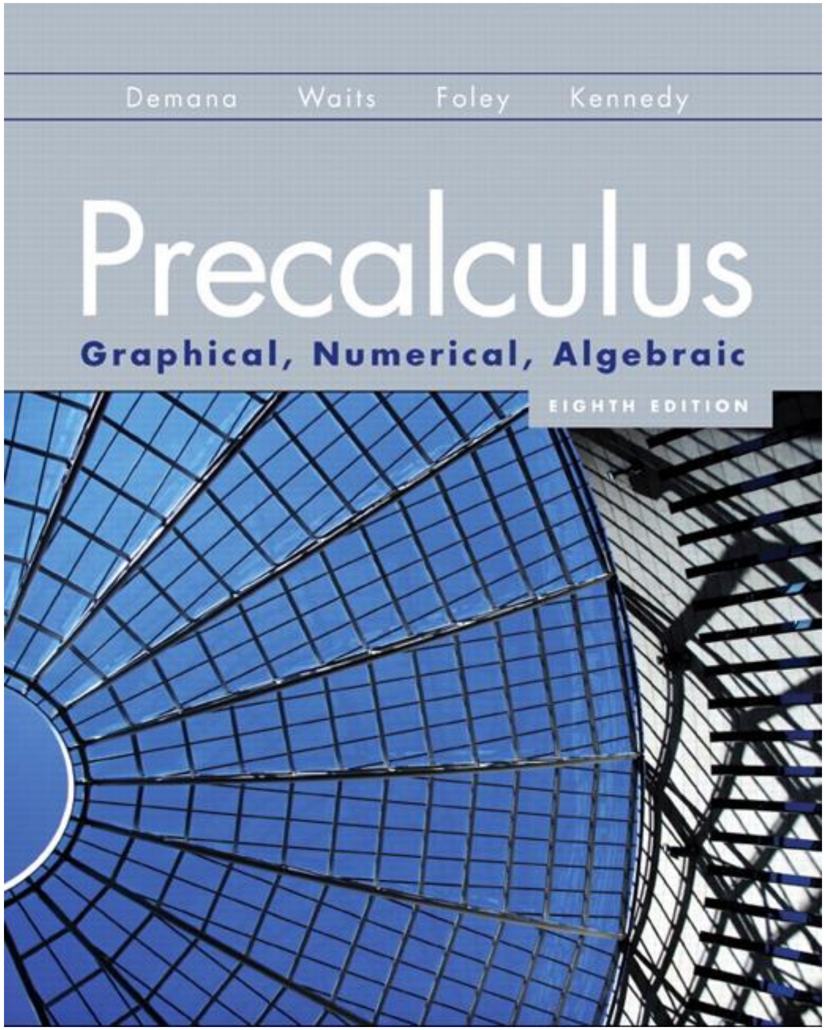
P.6

Complex Numbers



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What you'll learn about

- Complex Numbers
- Operations with Complex Numbers
- Complex Conjugates and Division
- Complex Solutions of Quadratic Equations

… and why

The zeros of polynomials are complex numbers.

Complex Number

A **complex number** is any number that can be written in the form

$$a + bi,$$

where a and b are real numbers. The real number a is the **real part**, the real number b is the **imaginary part**, and $a + bi$ is the **standard form**.

Addition and Subtraction of Complex Numbers

If $a + bi$ and $c + di$ are two complex numbers, then

Sum: $(a + bi) + (c + di) = (a + c) + (b + d)i$,

Difference: $(a + bi) - (c + di) = (a - c) + (b - d)i$.

Example **Multiplying Complex Numbers**

Find $(3+2i)(4-i)$.

Solution

Find $(3+2i)(4-i)$.

$$\begin{aligned}&(3+2i)(4-i)\\&=12-3i+8i-2i^2\\&=12+5i-2(-1)\\&=12+5i+2\\&=14+5i\end{aligned}$$

Complex Conjugate

The **complex conjugate** of the complex number

$$z = a + bi$$

is

$$\bar{z} = \overline{a + bi} = a - bi.$$

Complex Numbers

The **multiplicative identity** for the complex numbers is $1 = 1 + 0i$.

The **multiplicative inverse**, or **reciprocal**, of $z = a + bi$ is

$$z^{-1} = \frac{1}{z} = \frac{1}{a+bi} = \frac{1}{a+bi} \cdot \frac{a-bi}{a-bi} = \frac{a}{a^2+b^2} - \frac{b}{a^2+b^2}i$$

Example **Dividing Complex Numbers**

Write the complex number in standard form.

$$\frac{3}{5-i}$$

$$\frac{2+i}{3-2i}$$

Solution

□□□□□ the complex number in standard form.

$$\frac{3}{5-i} = \frac{3}{5-i}\cdot\frac{5+i}{5+i}$$

$$= \frac{15+3i}{5^2+1^2}$$

$$= \frac{15}{26}+\frac{3}{26}i$$

$$\frac{2+i}{3-2i} = \frac{2+i}{3-2i}\cdot\frac{3+2i}{3+2i}$$

$$= \frac{6+4i+3i+2i^2}{3^2+2^2}$$

$$= \frac{4+7i}{13}$$

$$= \frac{4}{13}+\frac{7}{13}i$$

Discriminant of a Quadratic Equation

For a quadratic equation $ax^2 + bx + c = 0,$ where $a, b,$ and c are real numbers and $a \neq 0.$

- if $b^2 - 4ac > 0,$ there are two distinct real solutions.
- if $b^2 - 4ac = 0,$ there is one repeated real solution.
- if $b^2 - 4ac < 0,$ there is a complex pair of solutions.

Example **Solving a Quadratic Equation**

Solve $3x^2 + 4x + 5 = 0.$

Example **Solving a Quadratic Equation**

Solve $3x^2 + 4x + 5 = 0.$

$a = 3,\ \ b = 4,$ and $c = 5$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(3)5}}{2(3)} = \frac{-4 \pm \sqrt{-44}}{6} = \frac{-4 \pm 2i\sqrt{11}}{6}$$

So the solutions are $x = -\frac{2}{3} - \frac{\sqrt{11}}{3}i$ and $x = -\frac{2}{3} + \frac{\sqrt{11}}{3}i.$

Quick Review

Add or subtract, and simplify.

1. $(2x+3)+(-x+3)$

2. $(4x-3)-(x+4)$

Multiply and simplify.

3. $(x+3)(x-2)$

4. $\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)$

5. $(2x+1)(3x+5)$

Quick Review Solutions

Add or subtract, and simplify.

1. $(2x+3)+(-x+3)$ $\quad x+6$

2. $(4x-3)-(x+4)$ $\quad 3x-7$

Multiply and simplify.

3. $(x+3)(x-2)$ $\quad x^2+x-6$

4. $\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)$ $\quad x^2-3$

5. $(2x+1)(3x+5)$ $\quad 6x^2+13x+5$