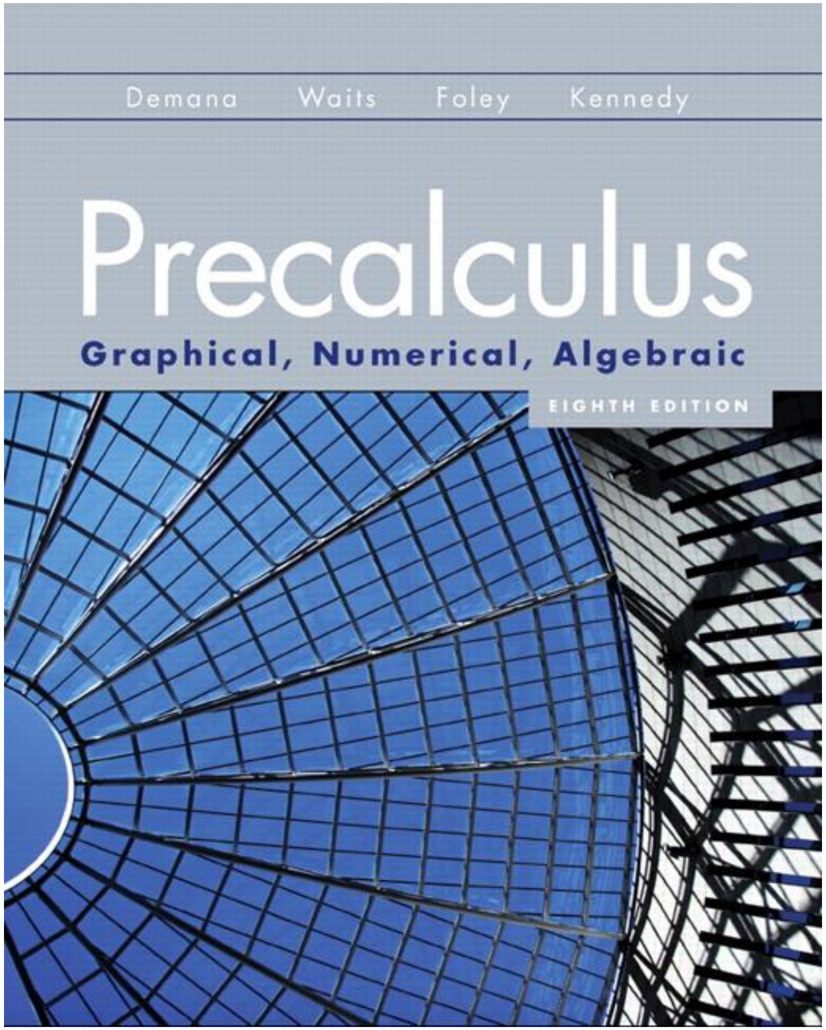
P.5

Solving Equations Graphically, Numerically and Algebraically



What you'll learn about

- Solving Equations Graphically
- Solving Quadratic Equations
- Approximating Solutions of Equations Graphically
- Approximating Solutions of Equations Numerically with Tables
- Solving Equations by Finding Intersections

… and why

These basic techniques are involved in using a graphing utility to solve equations in this textbook.

Example **Solving by Finding *x*-Intercepts**

Solve the equation $2x^2 - 3x - 2 = 0$ graphically.

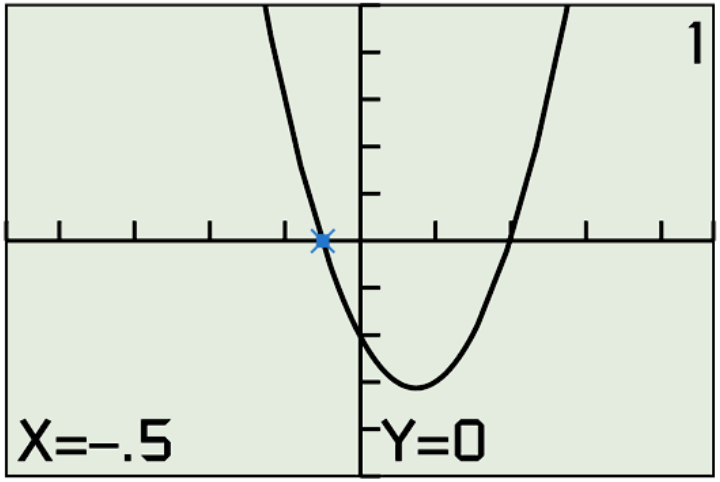
Solution

Solve the equation $2x^2 - 3x - 2 = 0$ graphically.

Find the x-intercepts of $y = 2x^2 - 3x - 2$.

Use the Trace to see that $(-0.5, 0)$ and $(2,0)$ are x-intercepts.

Thus the solutions are $x = -0.5$ and $x = 2$.



[–4.7, 4.7] by [–5, 5]

Zero Factor Property

Let a and b be real numbers.

If $ab = 0$, then $a = 0$ or $b = 0$.

Quadratic Equation in *x*

A **quadratic equation in *x*** is one that can be written in the form

$$ax^2 + bx + c = 0,$$

where a, b, and c are real numbers with $a \neq 0$.

Completing the Square

To solve $x^2 + bx = c$ by **completing the square**, add $(b/2)^2$ to both sides of the equation and factor the left side of the new equation.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$$

$$\left(x + \frac{b}{2}\right)^2 = c + \frac{b^2}{4}$$

Quadratic Formula

The solutions of the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the **quadratic formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Example **Solving Using the Quadratic Formula**

Solve the equation $2x^2 + 3x - 5 = 0.$

Solution

Solve the equation $2x^2 + 3x - 5 = 0.$

$$a = 2,\ b = 3,\ c = -5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4(2)(-5)}}{2(2)}$$

$$= \frac{-3 \pm \sqrt{49}}{4} = \frac{-3 \pm 7}{4}$$

$$x = -\frac{5}{2} \text{ or } x = 1.$$

Solving Quadratic Equations Algebraically

These are four basic ways to solve quadratic equations algebraically.

1. Factoring
2. Extracting Square Roots
3. Completing the Square
4. Using the Quadratic Formula

Agreement about Approximate Solutions

For applications, round to a value that is reasonable for the context of the problem. For all others round to two decimal places unless directed otherwise.

Example **Solving by Finding Intersections**

Solve the equation $-2|x-2| = -3.$

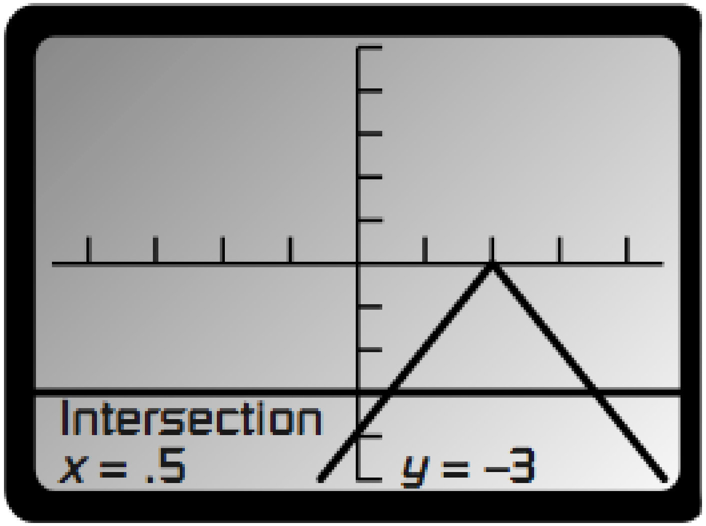
Solution

Solve the equation $-2|x-2|=-3.$

Graph $y=-2|x-2|$ and $y=-3.$

Use Trace or the intersect feature of your grapher to find the points of intersection. The graph indicates that the solutions are

$x=0.5$ and $x=3.5.$



$[-4.7, 4.7]$ by $[-5, 5]$

Quick Review

Expand the product.

1. $(x+2y)^2$

2. $(2x+1)(4x-3)$

Factor completely.

3. x^3+2x^2-x-2

4. y^4+5y^2-36

5. Combine the fractions and reduce the resulting fraction to lowest terms. $\dfrac{x}{2x+1}-\dfrac{2}{x-1}$

Quick Review Solutions

Expand the product.

1. $(x+2y)^2$ $\quad x^2+4xy+4y^2$

2. $(2x+1)(4x-3)$ $\quad 8x^2-2x-3$

Factor completely.

3. x^3+2x^2-x-2 $\quad (x+1)(x-1)(x+2)$

4. y^4+5y^2-36 $\quad (y^2+9)(y-2)(y+2)$

5. Combine the fractions and reduce the resulting fraction to lowest terms. $\dfrac{x}{2x+1}-\dfrac{2}{x-1}$ $\quad \dfrac{x^2-5x+2}{(2x+1)(x-1)}$