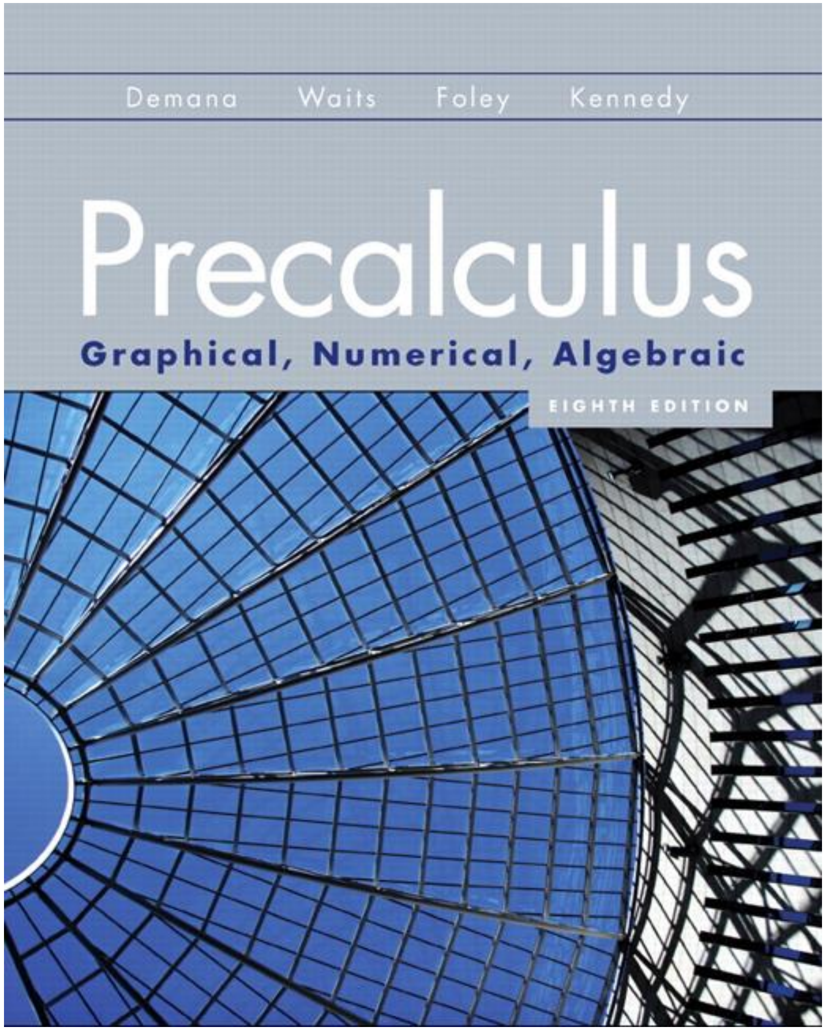
P.4

Lines in the Plane



What you'll learn about

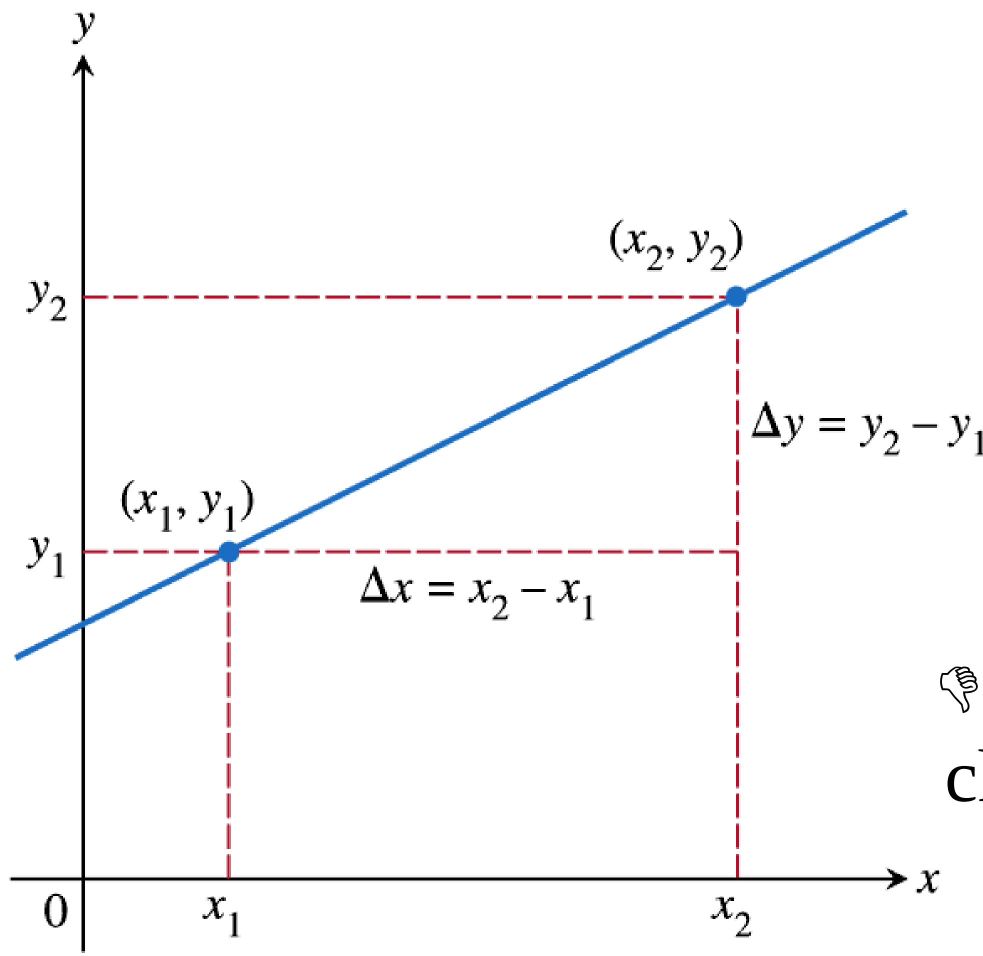
- Slope of a Line
- Point-Slope Form Equation of a Line
- Slope-Intercept Form Equation of a Line
- Graphing Linear Equations in Two Variables
- Parallel and Perpendicular Lines
- Applying Linear Equations in Two Variables

… and why

Linear equations are used extensively in applications involving business and behavioral science.

Slope of a Line

Δy "delta y"
change in y



Δx "delta x"
change in x

Slope of a Line

The slope of the nonvertical line through the points (x_1, y_1) and (x_2, y_2) is $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$.

If the line is vertical, then $x_1 = x_2$ and the slope is undefined.

Example **Finding the Slope of a Line**

Find the slope of the line containing the points (3, –2) and (0, 1).

Example **Finding the Slope of a Line**

Find the slope of the line containing the points (3, –2) and (0, 1).

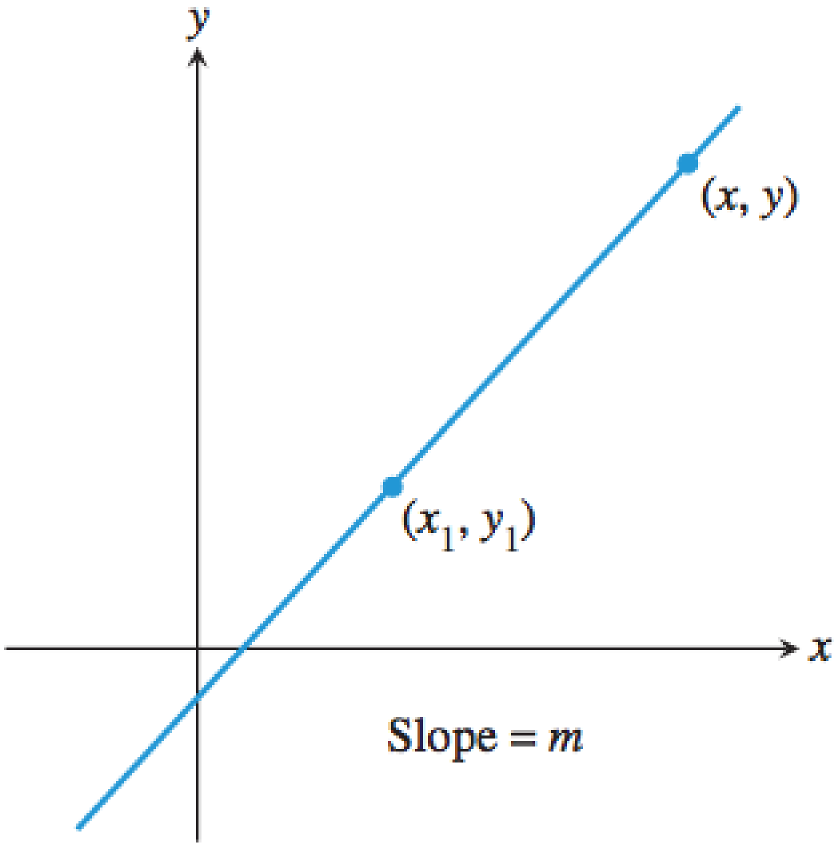
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-2)}{0-3} = \frac{3}{-3} = -1$$

Thus, the slope of the line is -1.

Point-Slope Form of an Equation of a Line

The **point - slope form** of an equation of a line that passesthrough the point (x_1, y_1) and has slope m is

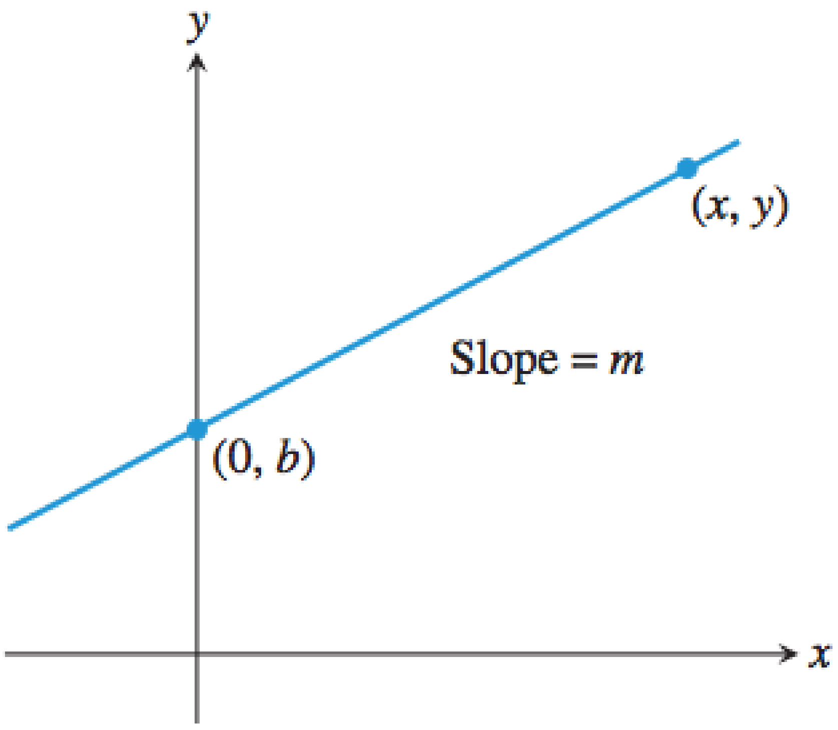
$$y - y_1 = m(x - x_1).$$



Slope-Intercept Form of an Equation of a Line

The **slope-intercept form** of an equation of a line with slope m and y-intercept $(0, b)$ is

$$y = mx + b.$$



Forms of Equations of Lines

General form:	$Ax + By + C = 0$, A and B not both zero
Slope-intercept form:	$y = mx + b$
Point-slope form:	$y - y_1 = m(x - x_1)$
Vertical line:	$x = a$
Horizontal line:	$y = b$

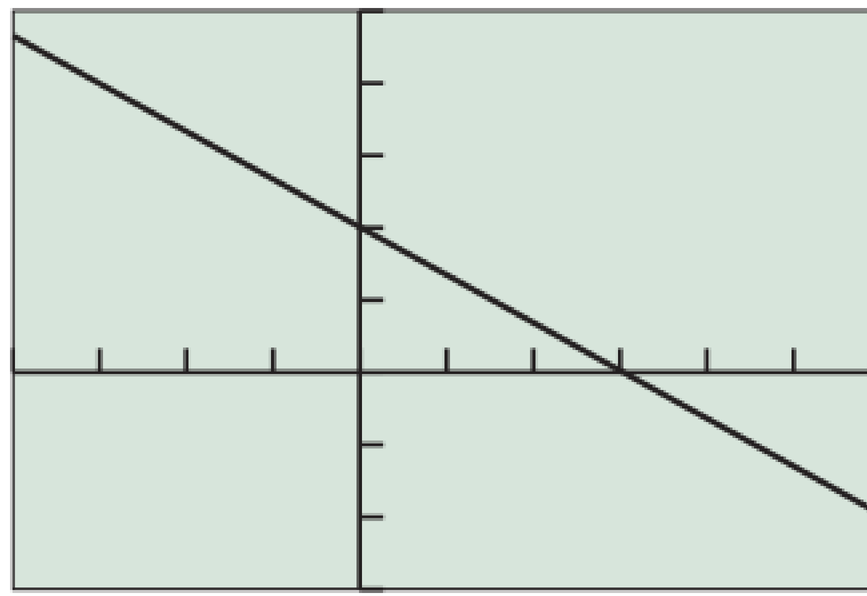
Graphing with a Graphing Utility

To draw a graph of an equation using a grapher:

1. Rewrite the equation in the form
   $y =$ (an expression in x).
2. Enter the equation into the grapher.
3. Select an appropriate viewing window.
4. Press the "graph" key.

Viewing Window

WINDOW
Xmin=–10
Xmax=10
Xscl=1
Ymin=–10
Ymax=10
Yscl=1
Xres=1



[–4, 6] by [–3, 5]

Parallel and Perpendicular Lines

1. Two nonvertical lines are parallel if and only if their slopes are equal.
2. Two nonvertical lines are perpendicular if and only if their slopes m_1 and m_2 are opposite reciprocals.

   That is, if and only if $m_1 = -\dfrac{1}{m_2}$.

Example **Finding an Equation of a Parallel Line**

Find an equation of a line through $(2,-3)$ that is parallel to $4x + 5y = 10$.

Solution

Find an equation of a line through $(2,-3)$ that is parallel to $4x + 5y = 10.$

Find the slope of $4x + 5y = 10.$

$$5y = -4x + 10$$

$$y = -\frac{4}{5}x + 2$$ The slope of this line is $-\frac{4}{5}$.

Use point-slope form:

$$y + 3 = -\frac{4}{5}(x - 2) \text{ so } y = -\frac{4}{5}x - \frac{7}{5}$$

Example **Finding an Equation of a Perpendicular Line**

Find an equation of a line through $P(-4,5)$ that is perpendicular to the line L with equation $2x - y = 1$.

Solution

Find an equation of a line through $P(-4,5)$ that is perpendicular to the line L with equation $2x - y = 1$.

Find the slope of $2x - y = 1$.

$$-y = -2x + 1$$

$y = 2x - 1$ Slope is 2.

Perpendicular slope is $-\frac{1}{2}$. Use point-slope form:

$$y - 5 = -\frac{1}{2}\left(x - (-2)\right) \text{ so } y = -\frac{1}{2}x + 3$$

Example **Finding a Linear Model**

American's disposable income in trillions of dollars is given in the table on the next slide.

(a) Write a linear equation for Americans' disposable income y in terms of the year x using the points (2002,8) and (2004,8.9).

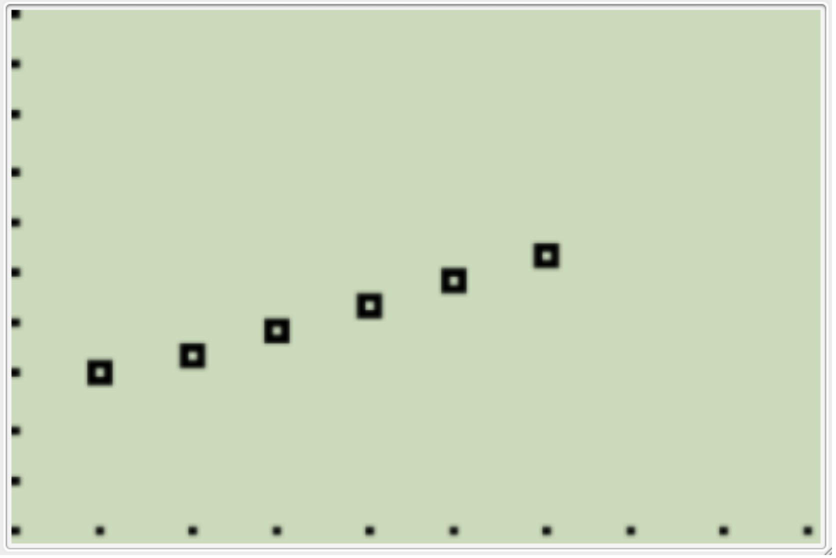
(b) Use the equation in (a) to estimate Americans' disposable income in 2005.

(c) Use the equation in (a) to predict Americans' disposable income in 2010.

(d) Superimpose a graph of the linear equation in (a) on a scatter plot of the data.

Example Finding a Linear Model

Year	Amount (trillions of dollars)
2002	8
2003	8.4
2004	8.9
2005	9.3
2006	9.9
2007	10.4



Solution

(a) Let $y = mx + b$. Find the slope $m \frac{8.9-8}{2004-2002} = 0.45$

Use (2002, 8) to find b.

$$y = 0.45x + b$$
$$8 = 0.45(2002) + b$$
$$b = 8 - 900.9 = -892.9$$

$$y = 0.45x - 892.9$$

Solution

(b) Find y when $x = 2005$.

$$y = 0.45x - 892.9$$

$$y = 0.45(2005) - 892.9$$

$$y = 9.35$$

So we estimate Americans' disposable income in 2005 to be 9.35 trillion dollars, a little more than the actual amount of 9.3 trillion dollars.

Solution

(c) Find y when $x = 2010$.

$$y = 0.45x - 892.9$$

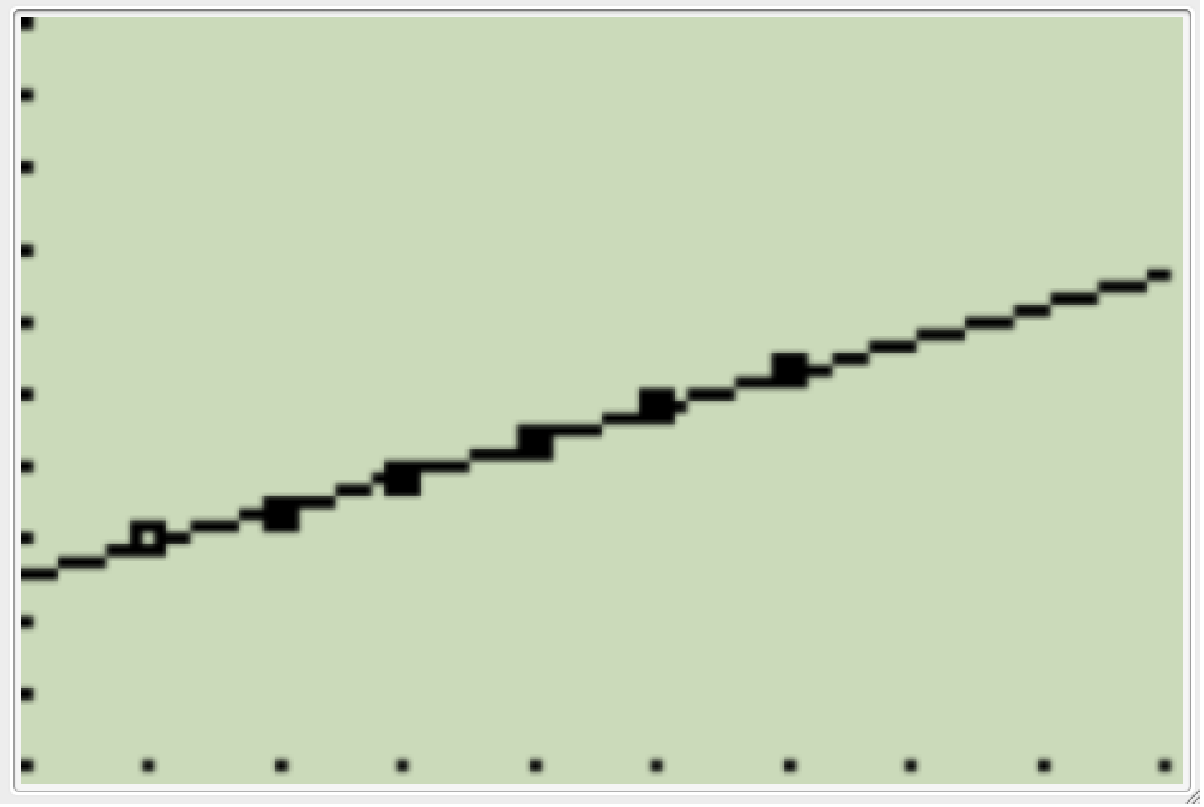
$$y = 0.45(2010) - 892.9$$

$$y = 11.6$$

So we predict Americans' disposable income in 2010 to be 11.6 trillion dollars.

Solution

(d) Here's the graph and scatter plot.



[2001,2010] by [5, 15]

Quick Review

Solve for *x*.

1. $-50x+100=200$

2. $3(1-2x)+4(x+2)=10$

Solve for *y*.

3. $2x\ -3y=5$

4. $2x\ -3(x+y)=y$

5. Simplify the fraction. $\dfrac{7-2}{-10-(-3)}$

Quick Review Solutions

Solve for *x*.

1. $-50x + 100 = 200$ $x = -2$

2. $3(1-2x) + 4(x+2) = 10$ $x = \frac{1}{2}$

Solve for *y*.

3. $2x - 3y = 5$ $y = \frac{2x-5}{3}$

4. $2x - 3(x+y) = y$ $y = \frac{-x}{4}$

5. Simplify the fraction. $\frac{7-2}{-10-(-3)}$ $-\frac{5}{7}$