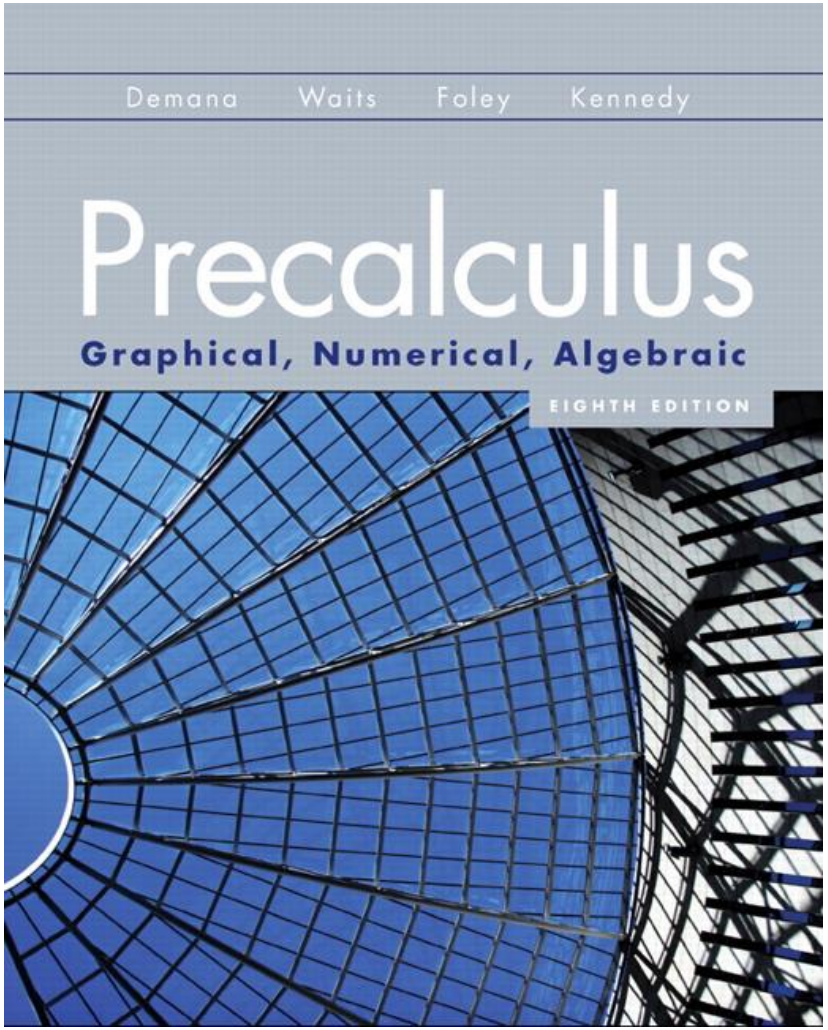
P.3

Linear Equations and Inequalities



What you'll learn about

- Equations
- Solving Equations
- Linear Equations in One Variable
- Linear Inequalities in One Variable

… and why

These topics provide the foundation for algebraic techniques needed throughout this textbook.

Properties of Equality

Let u, v, w, and z be real numbers, variables, or algebraic expressions.

1. Reflexive	$u = u$
2. Symmetric	If $u = v$, then $v = u$.
3. Transitive	If $u = v$, and $v = w$, then $u = w$.
4. Addition	If $u = v$ and $w = z$, then $u + w = v + z$.
5. Multiplication	If $u = v$ and $w = z$, then $uw = vz$.

Linear Equations in x

A **linear equation in x** is one that can be written in the form $ax + b = 0$, where a and b are real numbers with $a \neq 0$.

A **solution of an equation in x** is a value of x for which the equation is true. To **solve an equation in x** means to find all values of x for which the equation is true, that is, to find all solutions of the equation.

Operations for Equivalent Equations

An equivalent equation is obtained if one or more of the following operations are performed.

Operation	**Given Equation**	**Equivalent Equation**
1. Combine like terms, reduce fractions, and remove grouping symbols	$2x + x = \frac{3}{9}$	$3x = \frac{1}{3}$

Operations for Equivalent Equations

An equivalent equation is obtained if one or more of the following operations are performed.

Operation	**Given Equation**	**Equivalent Equation**
2. Perform the same operation on both sides.		
(a) Add (-3)	$x + 3 = 7$	$x = 4$
(b) Subtract $(2x)$	$5x = 2x + 4$	$3x = 4$
(c) Multiply by a nonzero constant (1/3)	$3x = 12$	$x = 4$
(d) Divide by a constant nonzero term (3)	$3x = 12$	$x = 4$

Example **Solving a Linear Equation Involving Fractions**

Solve for y. $\dfrac{10y-4}{4} = \dfrac{y}{4} + 2$

Solution

Solve for y. $\dfrac{10y-4}{4}=\dfrac{y}{4}+2$

$$\frac{10y-4}{4}=\frac{y}{4}+2$$

$$4\left(\frac{10y-4}{4}\right)=\left(\frac{y}{4}+2\right)4$$ Multiply by the LCD

$$10y-4=y+8$$ Distributive Property

$$9y=12$$ Simplify

$$y=\frac{4}{3}$$

Linear Inequality in x

A **linear inequality in x** is one that can be written in the form

$ax + b < 0,\ ax + b \le 0,\ ax + b > 0,$ or $ax + b \ge 0,$

where a and b are real numbers with $a \neq 0$.

Properties of Inequalities

Let $u, v, w,$ and z be real numbers, variables, or algebraic expressions, and c a real number.

1. Transitive If $u < v$, and $v < w$, then $u < w$.

2. Addition If $u < v$ then $u + w < v + w$.

If $u < v$ and $w < z$ then $u + w < v + z$.

3. Multiplication If $u < v$ and $c > 0$, then $uc < vc$.

If $u < v$ and $c < 0$, then $uc > vc$.

The above properties are true if $<$ is replaced by $\leq$.

There are similar properties for $>$ and $\geq$.

Example **Solving a Double Inequality**

Solve the inequality and graph its solution set.

$$\frac{5}{3} \geq \frac{2}{3} - \frac{1}{2}x > -\frac{4}{3}$$

Solution

Solve the inequality and graph its solution set.

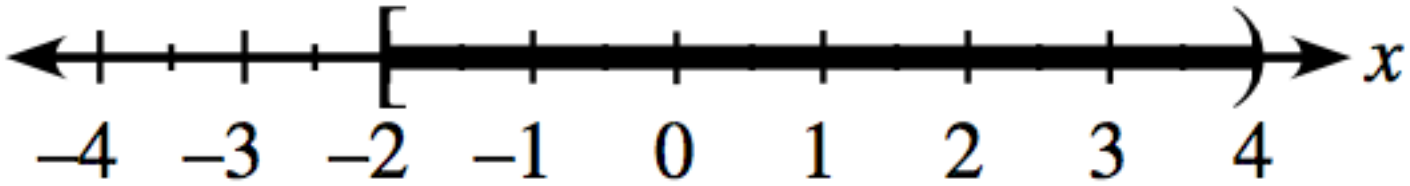
$$\frac{5}{3} \geq \frac{2}{3} - \frac{1}{2}x > -\frac{4}{3}$$

$$10 \geq 4 - 3x > -8$$

$$6 \geq -3x > -12$$

$$-2 \leq x < 4$$

$$[-2, 4)$$



Quick Review

Simplify the expression by combining like terms.

1. $2x + 4x - y - 2y - 3x$

2. $3(2x - 2) + 4(y - 1)$

Use the LCD to combine the fractions. Simplify the resulting fraction.

3. $\frac{3}{x} + \frac{4}{x}$

4. $\frac{x+2}{4} + \frac{x}{3}$

5. $2 + \frac{2}{y}$

Quick Review Solutions

Simplify the expression by combining like terms.

1. $2x+4x-y-2y-3x$ $\quad 3x-3y$

2. $3(2x-2)+4(y-1)$ $\quad 6x+4y-10$

Use the LCD to combine the fractions. Simplify the resulting fraction.

3. $\dfrac{3}{x}+\dfrac{4}{x}$ $\quad \dfrac{7}{x}$

4. $\dfrac{x+2}{4}+\dfrac{x}{3}$ $\quad \dfrac{7x+6}{12}$

5. $2+\dfrac{2}{y}$ $\quad \dfrac{2y+2}{y}$