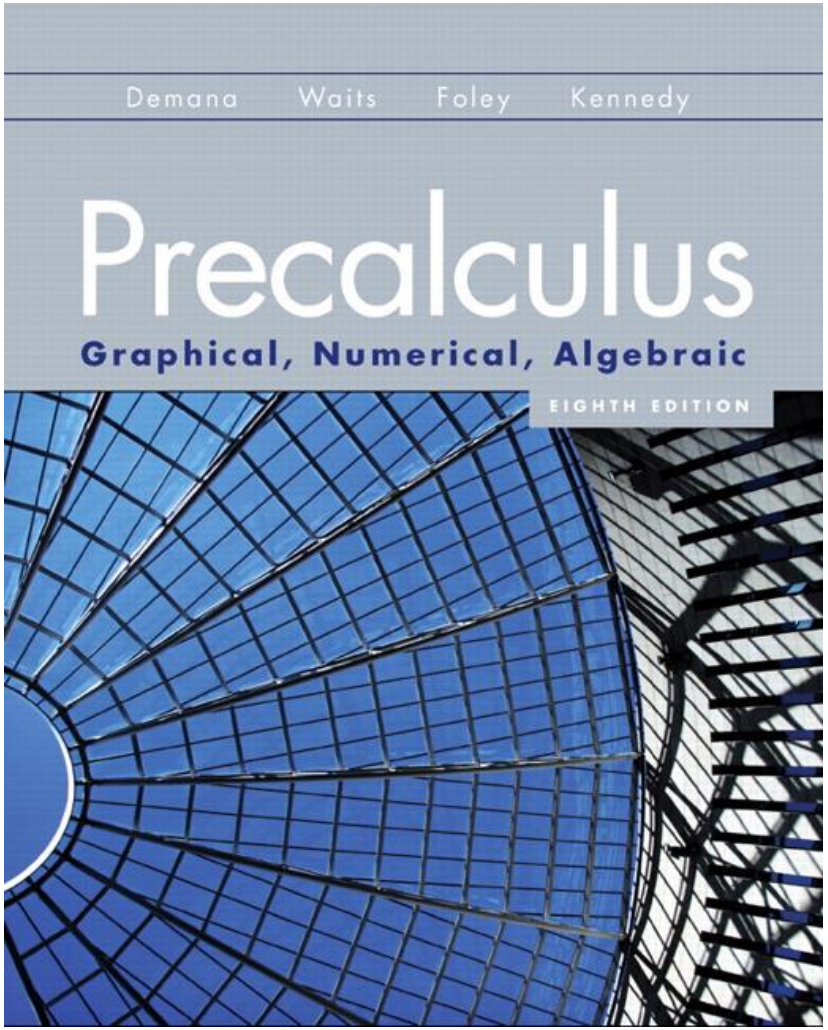
P.2

Cartesian Coordinate System



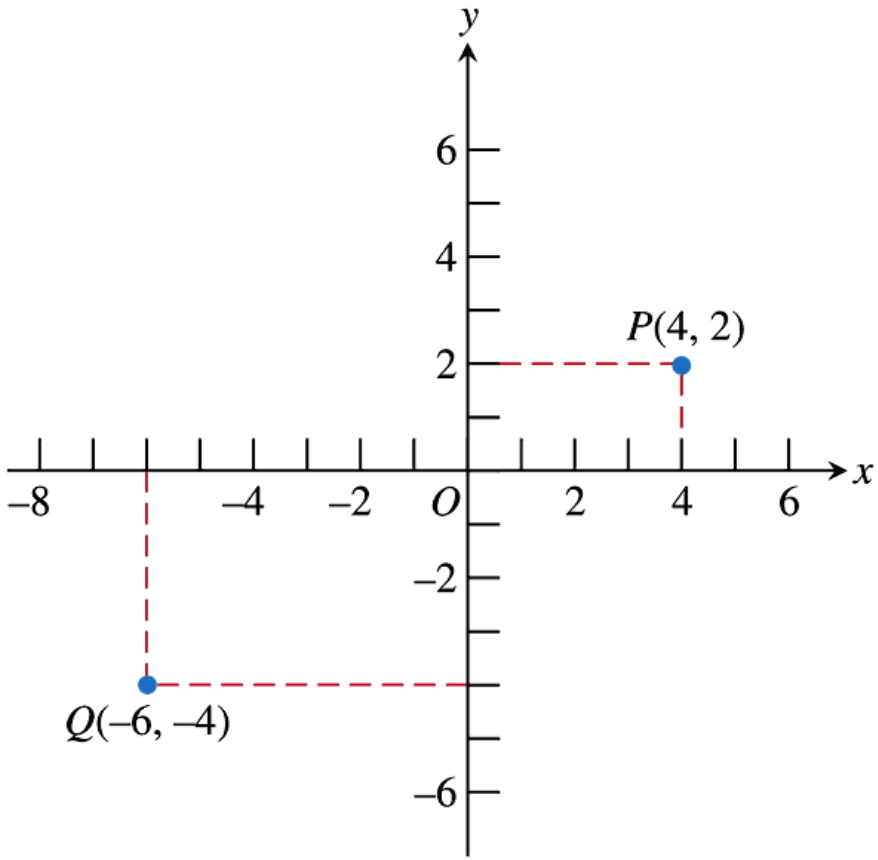
What you'll learn about

- Cartesian Plane
- Absolute Value of a Real Number
- Distance Formulas
- Midpoint Formulas
- Equations of Circles
- Applications

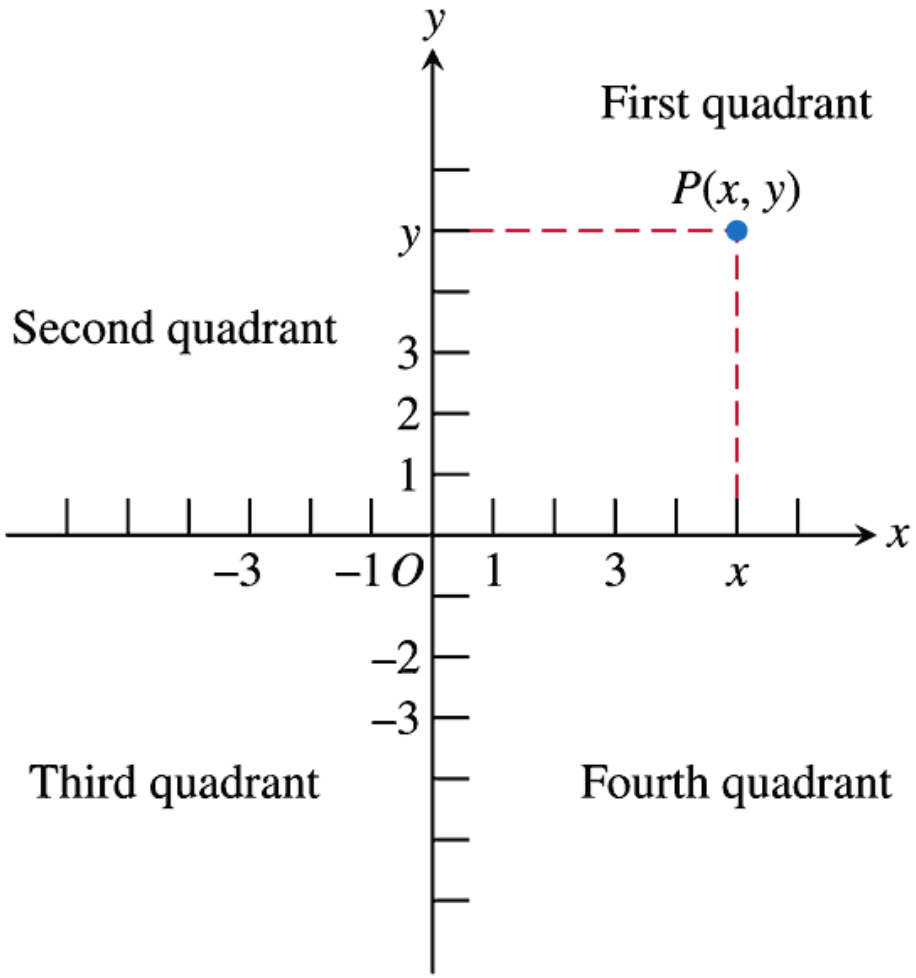
… and why

These topics provide the foundation for the material that will be covered in this textbook.

The Cartesian Coordinate Plane or Rectangular Coordinate System



Quadrants



Absolute Value of a Real Number

The **absolute value of a real number** a is

$$|a| = \begin{cases} a, & \text{if } a > 0 \\ -a & \text{if } a < 0. \\ 0, & \text{if } a = 0 \end{cases}$$

Example **Using the Definition of Absolute Value**

Evaluate:

$|-7|$

$|\pi - 5|$

Solution

Evaluate:

$$|-7| = 7$$

because $-7 < 0, |-4| = -(-7) = 7$

$$|\pi - 5| = -(\pi - 5) = 5 - \pi$$

because $\pi \approx 3.14, \pi - 5 < 0$

Properties of Absolute Value

Let a and b be real numbers.

1. $|a| \geq 0$
2. $|-a| = |a|$
3. $|ab| = |a||b|$
4. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, \; b \neq 0$

Distance Formula (Number Line)

Let a and b be real numbers.

The **distance between a and b** is $|a-b|$.

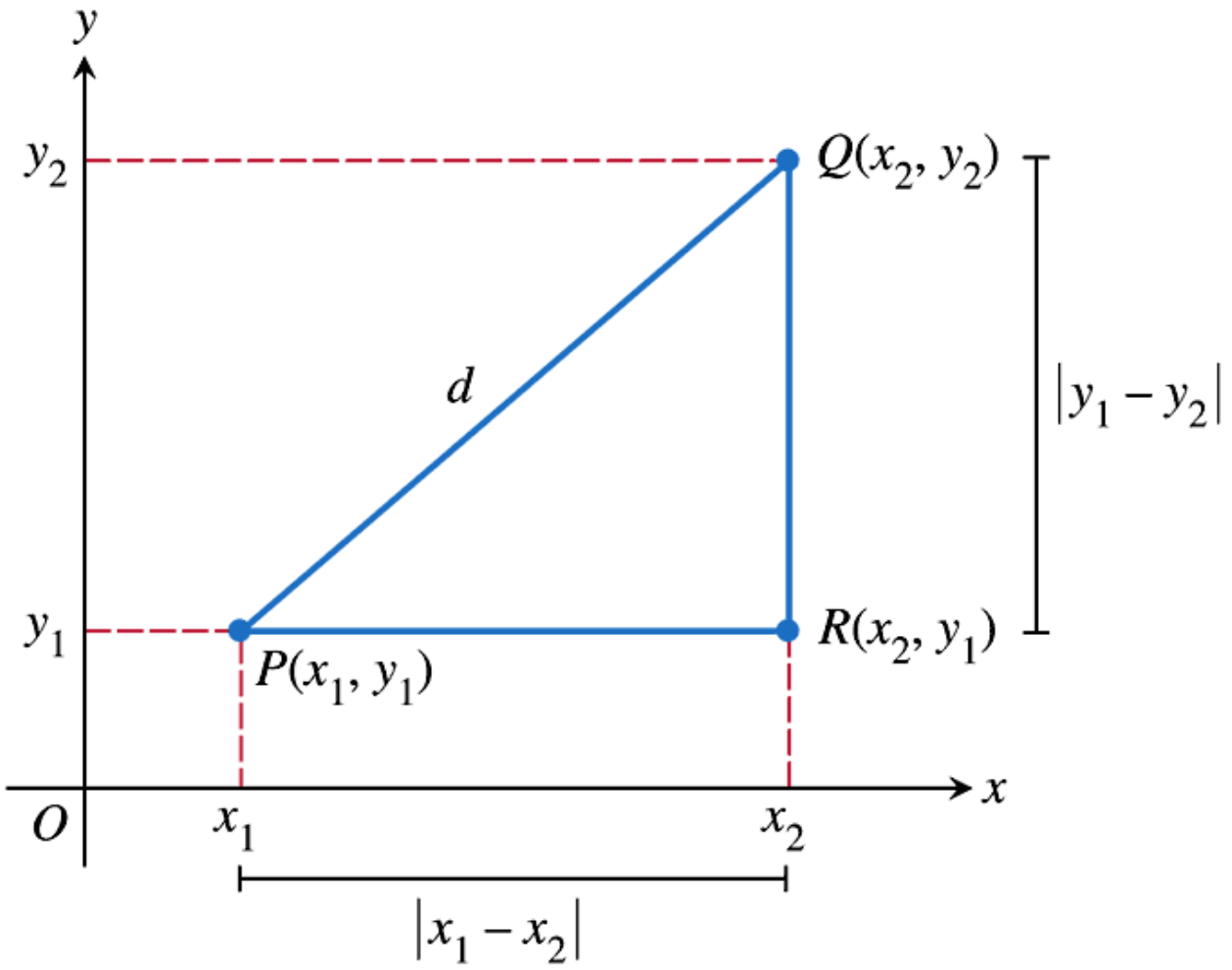
Note that $|a-b| = |b-a|$.

Distance Formula (Coordinate Plane)

The **distance d between points $P(x_1, y_1)$ and $Q(x_2, y_2)$** in the coordinate plane is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

The Distance Formula using the Pythagorean Theorem



Example **Finding the Distance Between Two Points**

P1 (2, 7) and P2 (5, 3)

Solution

P1 (2, 7) and P2 (5, 3)

$$d = \sqrt{(2-5)^2 + (7-3)^2}$$
$$= \sqrt{(-3)^2 + (4)^2}$$
$$= \sqrt{9+16}$$
$$= \sqrt{25}$$
$$= 5$$

Midpoint Formula (Number Line)

The **midpoint of the line segment with endpoints *a* and *b*** is

$$\frac{a+b}{2}.$$

Midpoint Formula (Coordinate Plane)

The **midpoint of the line segment with endpoints (a,b) and (c,d)** is

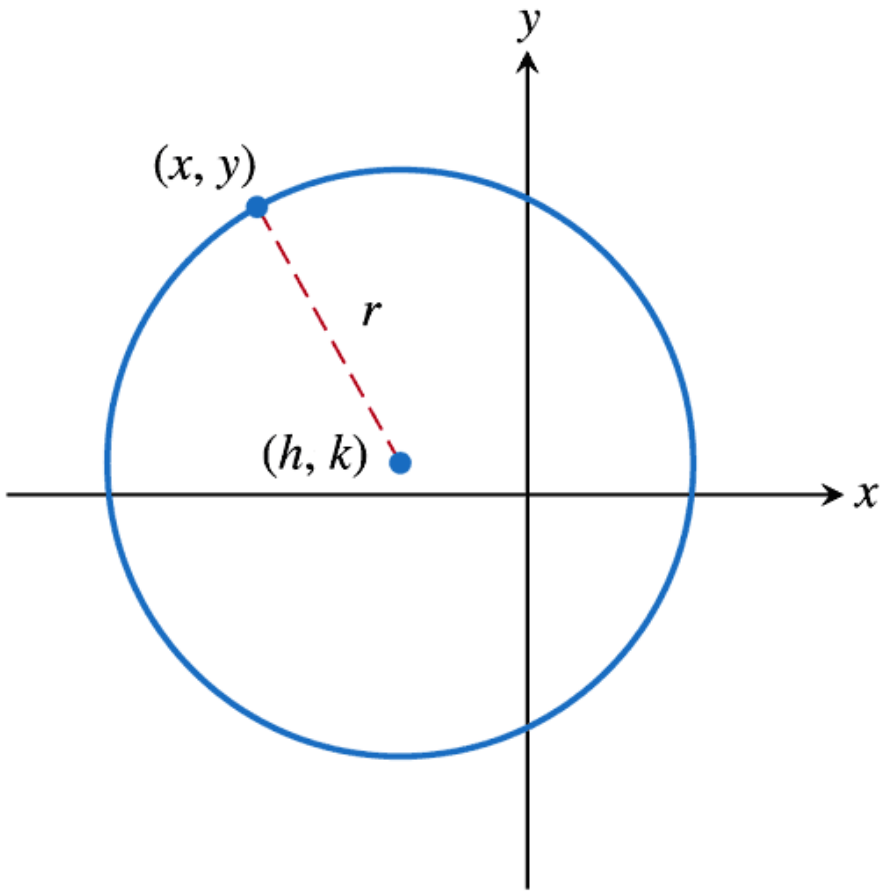
$$\left(\frac{a+c}{2}, \frac{b+d}{2}\right).$$

Standard Form Equation of a Circle

The **standard form equation of a circle** with center (h, k) and radius r is

$$(x-h)^2+(y-k)^2=r^2.$$

Standard Form Equation of a Circle



Example **Finding Standard Form Equations of Circles**

Find the standard form equation of the circlewith center $(2, -3)$ and radius 4.

Example **Finding Standard Form Equations of Circles**

Find the standard form equation of the circlewith center $(2, -3)$ and radius 4.

$(x-h)^2+(y-k)^2=r^2$ where $h=2,\ k=-3,$ and $r=4.$

Thus the equation is $(x-2)^2+(y+3)^2=16.$

Quick Review

1. Find the distance between $\frac{-5}{4}$ and $\frac{3}{2}$.

Use a calculator to evaluate the expression. Round answers to two decimal places.

2. $\sqrt{8^2+6^2}$

3. $\frac{-12+8}{2}$

4. $\sqrt{3^2+5^2}$

5. $\sqrt{(2-5)^2+(1-3)^2}$

Quick Review Solutions

1. Find the distance between $\frac{-5}{4}$ and $\frac{3}{2}$. 2.75

Use a calculator to evaluate the expression. Round answers to two decimal places.

2. $\sqrt{8^2+6^2}$ 10

3. $\frac{-12+8}{2}$ -2

4. $\sqrt{3^2+5^2}$ 5.83

5. $\sqrt{(2-5)^2+(1-3)^2}$ 3.61