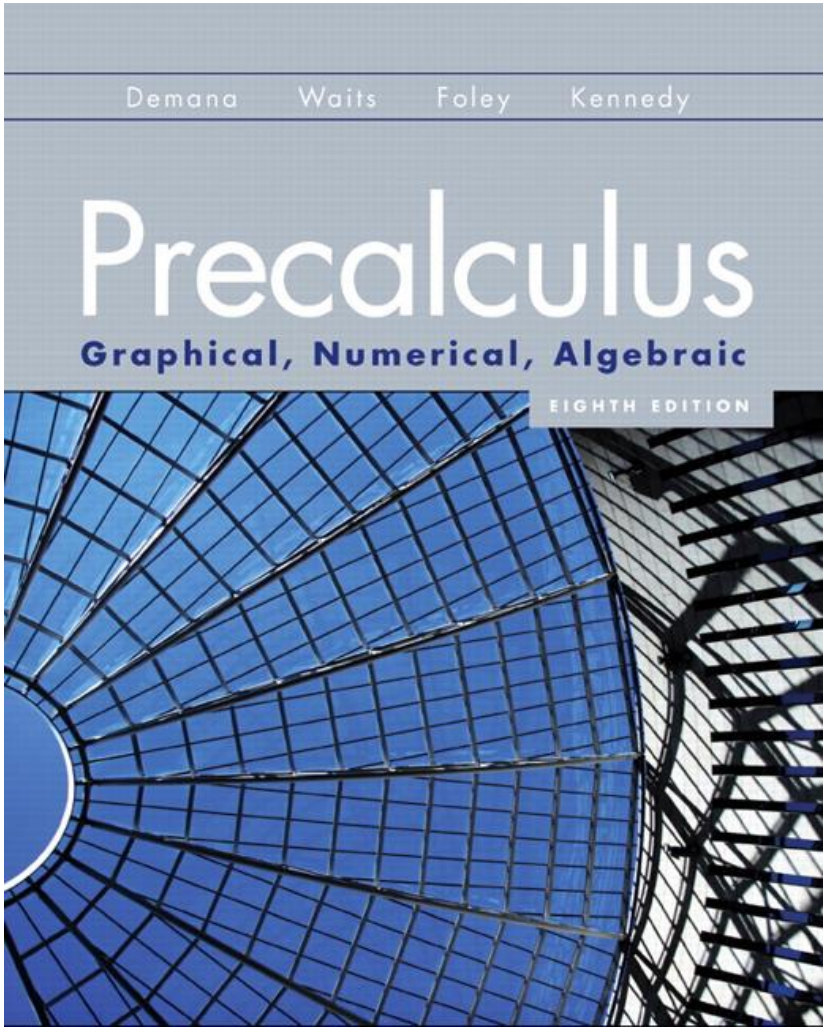
P.1

Real Numbers



What you'll learn about

- Representing Real Numbers
- Order and Interval Notation
- Basic Properties of Algebra
- Integer Exponents
- Scientific Notation

… and why

These topics are fundamental in the study of mathematics and science.

Real Numbers

A **real number** is any number that can be written as a decimal.

Subsets of the real numbers include:

- The **natural (or counting) numbers**: {1,2,3…}
- The **whole numbers**: {0,1,2,…}
- The **integers**: {…,-3,-2,-1,0,1,2,3,…}

Rational Numbers

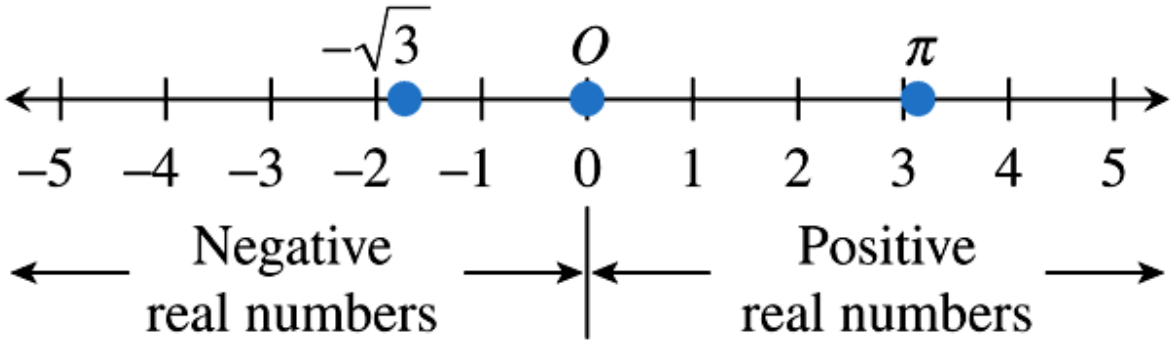
Rational numbers can be represented as a ratio a/b where a and b are integers and $b \neq 0$. We can describe rational numbers using **set-builder notation**:

$$\left\{ \frac{a}{b} \middle| a, b \text{ are integers, and } b \neq 0 \right\}$$

The decimal form of a rational number either **terminates** or is **indefinitely repeating**.

A number is **irrational** if it is *not* rational. The decimal form of an irrational number is infinitely nonrepeating.

The Real Number Line



Order of Real Numbers

Let a and b be any two real numbers.

Symbol	Definition	Read
$a > b$	$a - b$ is positive	a is greater than b
$a < b$	$a - b$ is negative	a is less than b
$a \geq b$	$a - b$ is positive or zero	a is greater than or equal to b
$a \leq b$	$a - b$ is negative or zero	a is less than or equal to b

The symbols $>$, $<$, $\geq$, and $\leq$ are **inequality symbols**.

Trichotomy Property

Let a and b be any two real numbers.

Exactly one of the following is true:

$$a < b, \quad a = b, \quad \text{or} \quad a > b.$$

Example **Interpreting Inequalities**

Describe and graph the interval of real numbers for $-3 \leq x < 5$.

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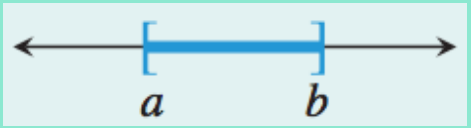
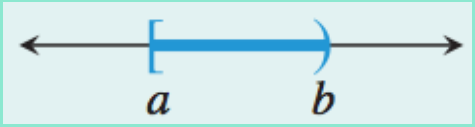
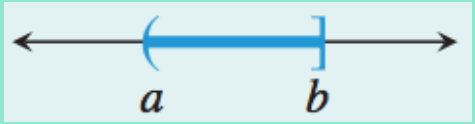
The *double inequality* represents all real numbers between –3 and 5, including –3 and excluding 5.

Bounded Intervals of Real Numbers

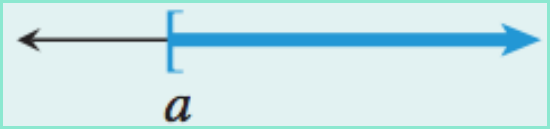
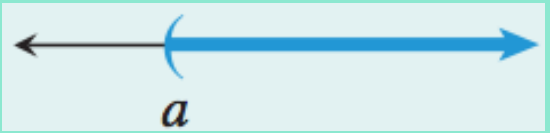
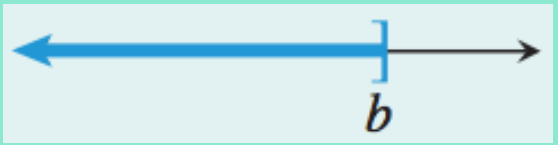
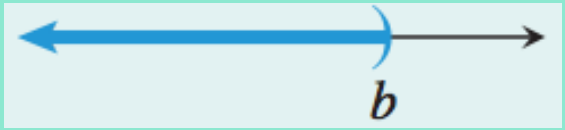
Let a and b be real numbers with $a < b$.

Interval Notation	**Inequality Notation**	**Graph**
$[a, b]$	$a \le x \le b$	
(a, b)	$a < x < b$	
$[a, b)$	$a \le x < b$	
$(a, b]$	$a < x \le \mathrm{b}$	

The numbers a and b are the **endpoints** of each interval.

Unbounded Intervals of Real Numbers

Let a and b be real numbers.

Interval Notation	Inequality Notation	Graph
$[a, \infty)$	$x \geq a$	
(a, ∞)	$x > a$	
$(-\infty, b]$	$x \leq b$	
$(-\infty, b)$	$x < b$	

Each of these intervals has exactly one endpoint, namely a or b.

Properties of Algebra

Let u, v, and w be real numbers, variables, or algebraic expressions.

1. Communative Property

Addition: $u + v = v + u$

Multiplication $uv = vu$

2. Associative Property

Addition: $(u + v) + w = u + (v + w)$

Multiplication: $(uv)w = u(vw)$

3. Identity Property

Addition: $u + 0 = u$

Multiplication: $u \cdot 1 = u$

Properties of Algebra

Let u, v, and w be real numbers, variables, or algebraic expressions.

4. Inverse Property

Addition: $u + (-u) = 0$

Mulitiplication: $u \cdot \frac{1}{u} = 1, \ u \neq 0$

5. Distributive Property

Multiplication over addition:

$u(v + w) = uv + uw$

$(u + v)w = uw + vw$

Multiplication over subtraction:

$u(v - w) = uv - uw$

$(u - v)w = uw - vw$

Example **Using the Distributive Property**

Write the expanded form of $(x + 3y)x$.

Write the factored form of $4z^2 + 20z$.

Example **Using the Distributive Property**

Write the expanded form of $(x + 3y)x$.

$$(x + 3y)x = x \cdot x + 3y \cdot x = x^2 + 3xy$$

Write the factored form of $4z^2 + 20z$.

$$4z^2 + 20z = 4z(z + 5)$$

Properties of the Additive Inverse

Let u, v, and w be real numbers, variables, or algebraic expressions.

Property	**Example**
1. $-(-u) = u$	$-(-3) = 3$
2. $(-u)v = u(-v) = -uv$	$(-4)3 = 4(-3) = -12$
3. $(-u)(-v) = uv$	$(-6)(-7) = 42$
4. $(-1)u = -u$	$(-1)5 = -5$
5. $-(u+v) = (-u)+(-v)$	$-(7+9) = (-7)+(-9) = -16$

Exponential Notation

Let a be a real number, variable, or algebraic expression and n a positive integer. Then $a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{\text{n factors}}$, where n is the **exponent**, a is the **base**, and a^n is the ***n*th power of *a***, read as "a to the nth power."

Properties of Exponents

Let u and v be a real numbers, variables, or algebraic expressions and m and n be integers. All bases are assumed to be nonzero.

Property	**Example**
1. $u^m u^n = u^{m+n}$	$5^3 \cdot 5^4 = 5^{3+4} = 5^7$
2. $\dfrac{u^m}{u^n} = u^{m-n}$	$\dfrac{x^9}{x^4} = x^{9-4} = x^5$
3. $u^0 = 1$	$8^0 = 1$
4. $u^{-n} = \dfrac{1}{u^n}$	$y^{-3} = \dfrac{1}{y^3}$
5. $(uv)^m = u^m v^m$	$(2z)^5 = 2^5 z^5 = 32z^5$
6. $(u^m)^n = u^{mn}$	$(x^2)^3 = x^{2 \cdot 3} = x^6$
7. $\left(\dfrac{u}{v}\right)^m = \dfrac{u^m}{v^m}$	$\left(\dfrac{a}{b}\right)^7 = \dfrac{a^7}{b^7}$

Example **Simplifying Expressions Involving Powers**

Simplify $\dfrac{u^2 v^{-3}}{u^{-1} v^2}$.

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Simplify $\dfrac{u^2v^{-3}}{u^{-1}v^2}$.

$$\frac{u^2v^{-3}}{u^{-1}v^2} = \frac{u^2u^1}{v^2v^3} = \frac{u^3}{v^5}$$

Scientific Notation

Any positive number can be written in **scientific notation**.

$c \times 10^m$, where $1 \leq c < 10$ and m is an integer.

Example **Converting to Scientific Notation**

Convert 0.0000345 to scientific notation.

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$$0.0000345 = 3.45 \times 10^{-5}$$

Example **Converting from Scientific Notation**

Convert 1.23×10^5 from scientific notation.

Example **Converting from Scientific Notation**

Convert 1.23×10^5 from scientific notation.

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Quick Review

1. List the positive integers between -4 and 4.

2. List all negative integers greater than -4.

3. Use a calculator to evaluate the expression $\frac{2(4.5)-3}{2.3-4.5}$. Round the value to two decimal places.

4. Evaluate the algebraic expression for the given values of the variable. $x^3+2x-1,\ x=-1,1.5$

5. List the possible remainders when the positive integer n is divided by 6.

Quick Review Solutions

1. List the positive integers between -4 and 4. $\{1,2,3\}$

2. List all negative integers greater than -4. $\{-3,-2,-1\}$

3. Use a calculator to evaluate the expression $\dfrac{2(4.5)-3}{2.3-4.5}$. Round the value to two decimal places. -2.73

4. Evaluate the algebraic expression for the given values of the variable. $x^3+2x-1,\ x=-1,1.5$ $\{-4,5.375\}$

5. List the possible remainders when the positive integer n is divided by 6. $1,2,3,4,5$